EMPIRICAL ORTHOGONAL FUNCTION REDUCTION OF A SIMPLE WIND-DRIVEN OCEAN CIRCULATION MODEL (Preliminary results)

Jairo Segura[1*], Thomas Frisius[1], Gualtiero Badin[2]

CliSAP Forschungsgruppe Dynamische Systeme, KlimaCampus Hamburg - Universität Hamburg[1]

Institute of Oceanography, University of Hamburg, Germany[2]

jairo.segura (at) zmaw.de[*]

INTRODUCTION

The oceanic circulation has an important influence on the Earth's climate. From the point of view of fluid dynamics, understanding the behavior of the global ocean circulation is an immensely complex problem. There are processes operating on spatial scales ranging from millimeters to tens of thousands of kilometers, and timescales from seconds to millennia. The fluid is rotating, stratified, turbulent, contained within a domain with a highly irregular boundary and subject to a forcing which is generally unpredictable in both space and time (Kiss, 2000). The most complex ocean circulation models include as much physics as can be computed but have an output almost as complex as the real system. On the other hand, there are highly idealized models which attempt to reduce the problem to the most relevant physics factors, so is still amenable to simple explanation (Scott, 1998). In this work, we focus on a simple model of barotropic wind-driven circulation in a homogeneous rectangular ocean.

MODEL

Taking specific cases (Böning, 1985) as a starting point, the nondimensional equation to solve is:

$$\frac{\partial}{\partial t}\nabla^2\psi' + \mathbf{RJ}(\psi',\nabla^2\psi') + {\psi'}_x = \mathrm{curl}_z\vec{\tau} + E_L\nabla^4\psi' - E_B\nabla^2\psi'$$

The windstress term is approximated as a double gire wind forcing (-sin2*y*), and the experiment analyzed is with lateral friction and with free slip boundary conditions

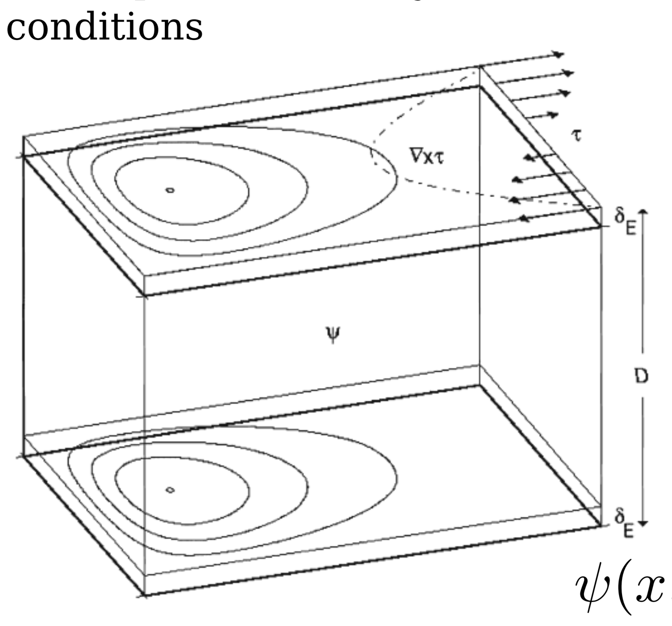

Figure 1: Sketch of the model domain (Dellnitz, 2000)

Rossby Number

$$R = \frac{\tau_0}{H\beta^2L^3}$$

Ekman numbers

$$E_L = \frac{A}{\beta L^3} \qquad E_B = \frac{K}{\beta L}$$

$$\psi(x,y,t) \sim \sum_{k=1}^{N}\sum_{l=1}^{N}\psi_{k,l}(t)\sin(kx)\sin(ly)$$

As a result for the chosen parameters, the experiment exhibits a non-steady state solution.

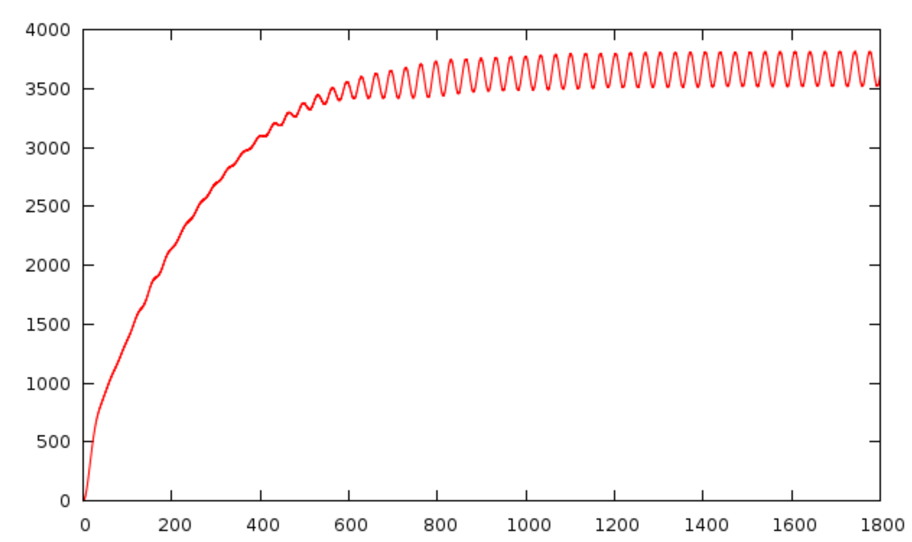

Figure 2: Kinetic energy against time, E_L=1x10^{-4}and R=1.4x10^{-3}.

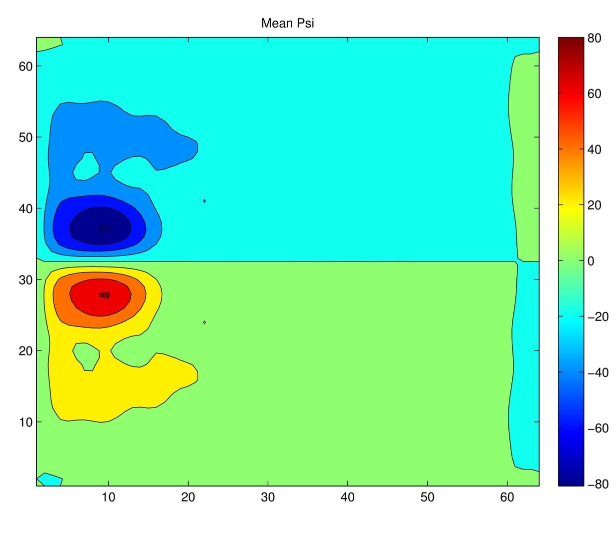

Figure 3: Mean streamfunction ψ_{max}=87.06

EOF REDUCTION METHOD

This kind of system can be analyzed applying an empirical orthogonal function analysis (EOF). Moreover the system can be reduced to a few equations, taking into account that the original model can be written as a dynamical system as follows

$$\frac{dX_i}{dt} = \sum_{j=1}^{N}\sum_{k=1}^{N} a_{ij}X_jX_k + \sum_{j=1}^{N} b_{ij}X_j + F_i(t)$$

where the variable X_i can be written in terms of the EOFs

$$X_i = \sum_{j=1}^{N} P_j(t)E_{ij}$$

the projection onto the EOFs yields to

$$\dot{a}_\nu = \frac{da_\nu}{dt} = \sum_{\mu=1}^{n_T}\sum_{\rho=1}^{n_T} N^q_{\nu\mu\rho}a_\mu a_\rho + \sum_{\mu=1}^{n_T} L^q_{\nu\mu}a_\mu + F^q_\nu(t)$$

which keeps the same structure as the original system.

RESULTS

Solving the new set of equations and comparing with the non-reduced model, the following graph is obtained.

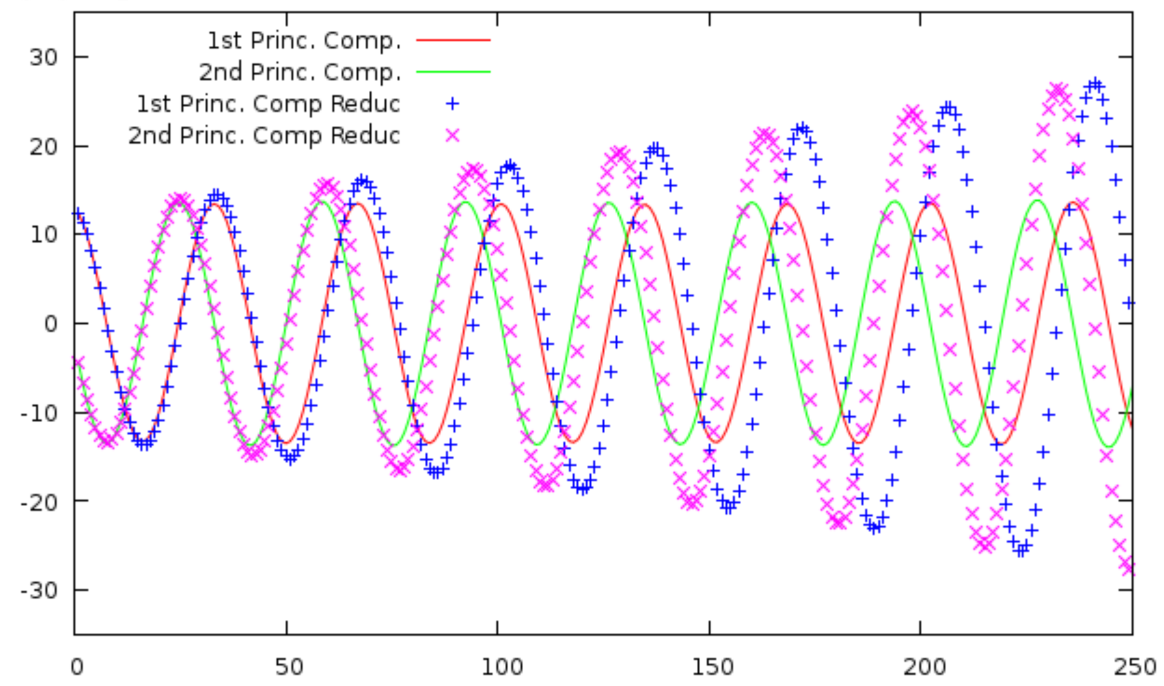

Figure 4: Comparison first and second principal components as a function of time

It can be observed that the differences with the full model grow, but applying the solution proposed by Achatz and Branstator (1998) adding an empirical based correction for the forcing and the linear operator, minimizes the error

$$\epsilon_t = \frac{\overline{\sum_{\nu=1}^{M}\left(\dot{a}^d_\nu - N^q_\nu - F_\nu - \sum_{\mu=1}^{M} L_{\nu\mu}a^d_\mu\right)^2}}{\overline{\sum_{\nu=1}^{M}(\dot{a}^d_\nu)^2}}$$

which leads to

$$\mathbf{L} = \mathbf{L}^e + \mathbf{L}^q = \overline{(\dot{\mathbf{a}}^d - \mathbf{N}^q)\mathbf{a}^{dT}}\;\overline{\mathbf{a}^d\mathbf{a}^{dT}}^{-1}$$

$$\mathbf{F} = \mathbf{F}^e + \mathbf{F}^q = -\overline{\mathbf{N}^q}$$

Solving the new set of equations, the results fits better to the non reduced model, using only the first 2 EOFs principal components.

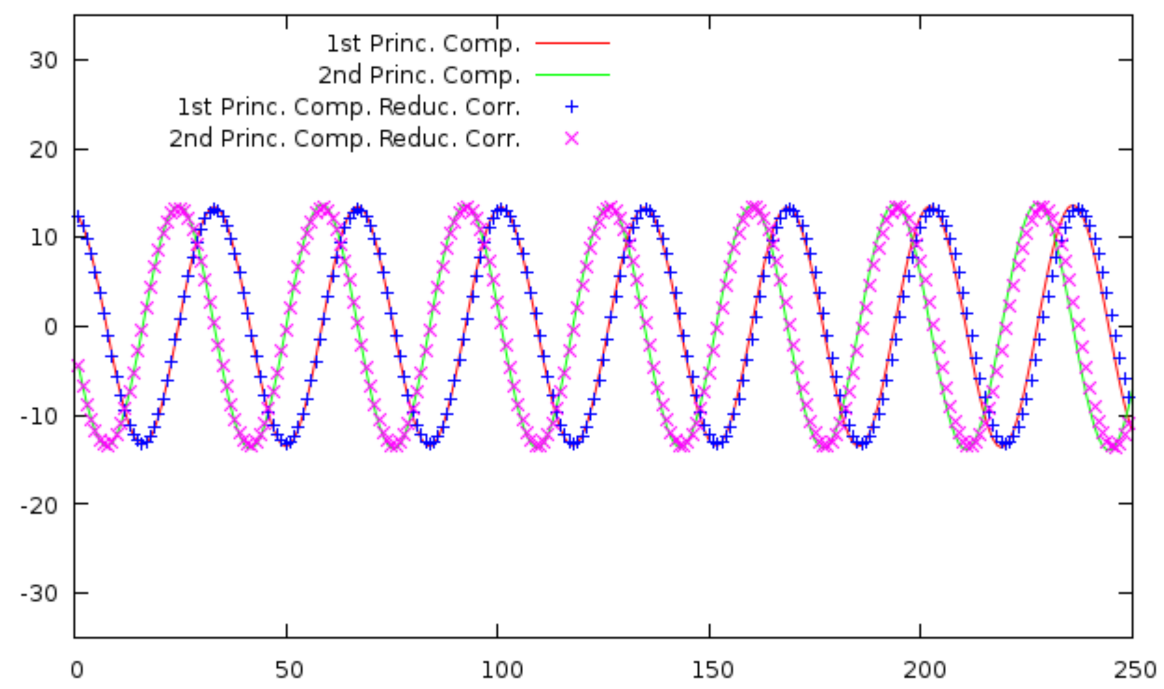

Figure 5: Comparison first and second principal components with correction as a function of time

With the correction, the results get more accurate, but despite a slight difference between the reduced model and the non-reduced one which could be assigned to the low number of EOFs used.

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DYNAMICAL SYSTEMS

KlimaCampus

Thomas Frisius
CliSAP Forschungsgruppe Dynamische Systeme
KlimaCampus, Universität Hamburg
Grindelberg 5, 20144 HamburgE-Post:
Thomas.Frisius@zmaw.de