A climate model of intermediate complexity: Physical consistence and applications

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<table>
<thead>
<tr>
<th>Why a mechanistic GCM, i.e., a GCM with intermediate complexity?</th>
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<tbody>
<tr>
<td>• Easily comprehensible, therefore allowing process studies by</td>
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<td>means of straightforward sensitivity experiments.</td>
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<td>• Development and testing of parameterizations/closures and</td>
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<td>may be applied also in complex models (e.g., subgrid-scale</td>
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<td>diffusion)</td>
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<td>• Good chance to achieve physical (or at least energetical)</td>
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<td>consistency.</td>
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<table>
<thead>
<tr>
<th>When using comprehensive (complex) GCMs?</th>
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<tbody>
<tr>
<td>• Detailed climate studies with realistic parameters.</td>
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<td>• Processes neglected in the mechanistic case matter.</td>
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Special focus of model development at IAP: Representation of gravity waves (GWs) and macro-turbulent diffusion

Reason:
GWs and turbulence are first-order dynamical processes in the mesosphere/lower thermosphere (MLT, 50-110 km)

Snapshot of temperature (K) and dissipation (K/d) around 85 km (January)
Lorenz energy cycle

- Frictional heating essential for entropy and energy budgets: net diabatic heating of the atmosphere = frictional heating (Lorenz, 1967)

- Molecular frictional heating occurs at the end of energy cascades through the mesoscales (including GWs) and turbulence
Conventional GCM with a diagnostic turbulence model

\[ d_t \mathbf{v} = -f \mathbf{e}_z \times \mathbf{v} - \frac{\nabla p}{\rho} + \rho^{-1} \partial_z (\rho K_z \partial_z \mathbf{v}) + \text{hyperdiffusion} \]

\[ d_t h = \frac{d_t p}{\rho} + Q_{rad} + Q_{lat} + \frac{c_p}{\rho} \partial_z (\rho \frac{T}{\Theta} K_z \partial_z \Theta) - \mathbf{v} \cdot \frac{\partial_z (\rho K_z \partial_z \mathbf{v})}{\rho}, \quad h = c_p T \]

- \( K_z \) and dynamic boundary conditions as usual
- \{hyperdiffusion\} without stress tensor; \(- \mathbf{v} \cdot \frac{\partial_z (\rho K_z \partial_z \mathbf{v})}{\rho}\) is flawed

Consistent formulation

\[ d_t \mathbf{v} = -f \mathbf{e}_z \times \mathbf{v} - \frac{\nabla p}{\rho} + \rho^{-1} \partial_z (\rho K_z \partial_z \mathbf{v}) + \rho^{-1} \nabla (\rho K_h S_h) \]

\[ d_t h = \frac{d_t p}{\rho} + Q_{rad} + Q_{lat} + \frac{c_p}{\rho} \partial_z (\rho \frac{T}{\Theta} K_z \partial_z \Theta) + \frac{K_z (\partial_z \mathbf{v})^2}{\partial_z} + K_h (S_h \nabla) \cdot \mathbf{v} \geq 0 \]

- \( S_h = S_h^T, \quad K_h (S_h \nabla) \cdot \mathbf{v} \geq 0, \) and \( S_h \mathbf{e}_z = 0 \) (Becker, 2001, JAS)
- no exchange of mechanical energy between the surface and the atmosphere \( \rightarrow \) energy conserving discretization of vertical momentum diffusion and shear production (finite-difference analogue of the no-slip condition)

(Burchard, 2002, OM; Becker, 2003, MWR; Boville & Bretherton, 2003, MWR)
Another issue: Heat diffusion based on potential temperature may violate the Second Law (Gassmann & Herzog, QJRMS, 2014)

Consistent formulation

\[ d_t \mathbf{v} = -f e_z \times \mathbf{v} - \frac{\nabla p}{\rho} + \rho^{-1} \partial_z (\rho K_z \partial_z \mathbf{v}) + \{\text{hyperdiffusion}\} \]

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- \( K_z \) and dynamic boundary conditions
- \{\text{hyperdiffusion}\}

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\[ d_t h = \frac{d_t p}{\rho} + Q_{rad} + Q_{lat} + \frac{c_P}{\rho} \partial_z (\rho \frac{T}{\Theta} K_z \partial_z \Theta) + \frac{K_z (\partial_z \mathbf{v})^2}{\text{geometric terms}} + K_h (S_h \nabla) \cdot \mathbf{v} \geq 0 \]

- \( S_h = S_h^T \), \( K_h (S_h \nabla) \cdot \mathbf{v} \geq 0 \), and \( S_h e_z = 0 \) (Becker, 2001, JAS)

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(Burchard, 2002, OM; Becker, 2003, MWR; Boville & Bretherton, 2003, MWR)
Kühlungsborn Mechanistic general Circulation Model (KMCM)

- standard spectral dynamical core; T120/L190 or T240/L190 up to 125 km; T330L100 up 60 km
- mechanistic because of simple representations of the diabatic heating rates due to radiation (temperature relaxation) and condensation

- hydrodynamically consistent parameterization of turbulent friction
  \[
  (\partial_t \tilde{v})_{\text{diff}} = \frac{1}{\rho} \nabla \left( \rho K_h \tilde{S} \right) + \frac{1}{\rho} \partial_z \left( \rho (K_z + \nu) \partial_z \tilde{v} \right), \quad \tilde{S} = \tilde{S}^T \quad \text{(ang. mom.)}
  \]

- diffusion coefficients adjust automatically to dynamic instability
  \[
  K_z = \left( l_z^2 \right) | \partial_z v | F(Ri), \quad K_h = \left( l_h^2 \right) | \tilde{S} | (1 + \alpha F(Ri))
  \]

- \( F(Ri) = \begin{cases} 
  \sqrt{1 - 18 Ri}, & Ri < 0 \\
  (1 + 9 Ri)^{-1}, & Ri \geq 0 
  \end{cases} \)

- \( Ri = N^2 / (\partial_z v)^2 \rightarrow Ri - 0.25 \) for \( p < 100 \) hPa

3 free parameters: 2 mixing lengths and \( \alpha \)
Simulated dynamically determined January climatology

Assumed equilibrium state for January
Gravity waves and planetary waves around the summer mesopause: Temperature at 0.003 hPa (~85 km) for different filtering of horizontal scales.
Snapshots of the simulated vertical wind in the summer hemisphere

**vertical wind (ms⁻¹)**

For higher resolution, the simulated gravity waves have smaller spatial scales (and higher frequencies).
Dependence of simulated GW scales on the spatial resolution

For higher resolution, the simulated gravity waves have smaller spatial scales (and higher frequencies). As a result, smaller GW amplitudes induce similar mean-flow effects.
Budget for the gravity-wave kinetic energy

\[ \partial_t \frac{\overline{v^2}}{2} = -\left( \left( \overline{v \cdot v} \right) \right) \cdot \overline{v} - \frac{\partial_z \overline{\rho w}}{\rho_r} - \frac{g \overline{\rho w}}{\rho_r} \text{ (adiabatic conversion)} - \frac{\left( \overline{w v} \right) \cdot \partial_z \overline{v}}{\rho_r} \text{ (vert. adv.)} + \overline{v \cdot R} \sim -\overline{\epsilon} \]
Spectral kinetic energy in an atmospheric GCM

Spectral representation of the horizontal wind:

\[
\mathbf{v} = - \sum_{n=1}^{N} \frac{a^2}{n(n+1)} \sum_{m=-n}^{+n} \left( \xi_{nm} (\mathbf{e}_z \times \nabla Y_{nm}) + D_{nm} \nabla Y_{nm} \right)
\]

\[
\mathbf{v} = \mathbf{v}^{\text{rot}} + \mathbf{v}^{\text{div}}
\]

Height-dependent global mean and spectral kinetic energy per unit mass:

\[
K = (4\pi)^{-1} \int \frac{\mathbf{v}^2}{2} \, d\Omega = \sum_{n=1}^{N} K_n
\]

\[
K_n = \frac{a^2}{8\pi n(n+1)} \sum_{m=-n}^{n} (\xi_{nm}^2 + D_{nm}^2)
\]

Analogously for enstrophy: \(Z_n = \frac{1}{8\pi} \sum_{m=-n}^{n} \xi_{nm}^2\)
Spectral budgets

\[
\dot{K}_n = \frac{a^2 / 4\pi}{n(n+1)} \sum_{m=-n}^{n} (\xi_{nm} \xi_{nm} + \dot{D}_{nm} D_{lnm}) , \quad \dot{Z}_n = \frac{1}{8\pi} \sum_{m=-n}^{n} \xi_{nm} \xi_{nm}
\]

compute \( \xi_{nm} \) and \( \dot{D}_{nm} \) for different contributions to the horizontal momentum equation via spectral transformation

→ spectral tendencies and spectral fluxes:

<table>
<thead>
<tr>
<th>HA</th>
<th>complete horizontal advection</th>
<th>( \dot{K}^{HA}_n )</th>
<th>( \sum_{\tilde{n}=n}^{N} \dot{K}^{HA}_{\tilde{n}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA2</td>
<td>(quasi-)2D horizontal advection (using just ( \mathbf{v}^{rot} ) instead of ( \mathbf{v} ))</td>
<td>( \dot{K}^{HA2}_n )</td>
<td>( \sum_{\tilde{n}=n}^{N} \dot{K}^{HA2}_{\tilde{n}} )</td>
</tr>
<tr>
<td>AC</td>
<td>adiabatic conversion</td>
<td>( \dot{Z}^{HA2}_n )</td>
<td>( \sum_{\tilde{n}=n}^{N} \dot{Z}^{HA2}_{\tilde{n}} )</td>
</tr>
<tr>
<td>VA</td>
<td>vertical advection</td>
<td>( \dot{\mathbf{v}} )</td>
<td>( \dot{\mathbf{v}} )</td>
</tr>
<tr>
<td>MD</td>
<td>momentum diffusion</td>
<td>( \dot{\mathbf{v}} )</td>
<td>( \dot{\mathbf{v}} )</td>
</tr>
</tbody>
</table>
Continuous spectral fluxes (T330L100)  
(Brune & Becker, 2013, JAS)

- horizontal enstrophy and energy cascades as expected
- spectral flux due to adiabatic conversion indicative of stratified turbulence
- almost no energy deposition by the mesoscales in the upper troposphere:
Consequences

- Stratified turbulence with the scaling \( I_z = I_h^{1/3} \epsilon^{1/3} / N \) is a physically motivated concept for the macro-turbulent behavior in the mesoscales above the boundary layer (possibly up to the lower thermosphere).

- Any macroturbulent diffusion for this inertial range must not only fulfill the conservation laws, but must in addition be scale-invariant.

- Such a closure has only recently been proposed (Schaefer-Rolffs, Knöpfel & Becker, Meteorol. Z, 2015; Schaefer-Rolffs & Becker, MWR, 2012 and 2015 subm.).
A new scale-invariant closure for truncation in the mesoscales

Diffusion coefficients for horizontal momentum

\[ K_h = l_h^2 |S_h| \gg K_z = l_z^2 |\partial_z v|, \quad l_z \propto \sqrt{l_h} \]

\[ S_h = \nabla \cdot v + (\nabla \cdot v)^T - \mathbf{1}(\nabla \cdot v) \]

\[ l_h^2 = \frac{1}{\left( \left( \frac{\Delta_h}{\Delta_h} \right)^2 |S_h| S_h - |S_h| |S_h| \right)} \]

Diffusion coefficients for any scalar variable \( C \) (sensible heat, tracer)

\[ K_h \rightarrow \frac{K_h}{\gamma_h}, \quad \frac{1}{\gamma_h} = \frac{l_{C,h}^2}{l_h^2} = \frac{1}{l_h^2} \left| \left( \frac{\Delta_h}{\Delta_h} \right)^2 |S_h| \nabla C - |S_h| \nabla C \right| \]

\[ K_z \rightarrow \frac{K_z}{\gamma_z}, \quad \frac{1}{\gamma_z} \propto \frac{l_{C,h}}{l_h} = \frac{1}{\sqrt{\gamma_h}} \]
Conventional Smagorinsky Model completed by hyperdiffusion

\[ \log_{10}(K_n (m^2 s^{-2})), \ 300-150 \ hPa \]

New DSM

\[ n^{-5/3} \]

\[ n^{-3} \]
KMCM as a mechanistic climate model

- Resolutions: T32L70, T42L110, or T120L190, all up to ~125 km

- Turbulent diffusion consistent with conservation laws and interacting with parameterized GWs


- Tracer transport based on the spectral transform method (Schlutow et al., 2014, JGR): Energy conserving tropospheric moisture cycle

- Continuous computation of radiative transfer from the surface up to the lower thermosphere, including deviations from LTE and the gray limit (Knöpfel & Becker, 2011, JQSRT; Becker et al., 2015, JASTP, subm.) → complete energy budget of the surface (slab ocean with prescribed oceanic heat-flux convergence; land-sea masks for surface heat capacity, albedo, and relative humidity)
Surface parameters and moisture cycle

- Heat capacity (MJm⁻²K⁻¹) & oceanic heating (Wm⁻²)
- Albedo & relative humidity (%)
- Annual-mean surface evaporation (Wm⁻²)
- Annual-mean latent heating (Wm⁻²)
Moisture cycle in the zonal mean

**radiative heating (Kd⁻¹)**

**latent plus turbulent heating (Kd⁻¹)**

**relative humidity (%)**

**([q*υ]+[g][v])/(q_∞) (ms⁻¹)**
DJF stationary waves

(a) DJF surface temperature (K)

(b) DJF zonal wind at 250 hPa (ms⁻¹)

(c) DJF z* at 5 hPa (m)
Zonal-mean climatology during July and January

(a) temperature (K) & res. mass streamf. (Mts⁻¹)

(b) zonal wind (ms⁻¹) & Div (EPF) (ms⁻¹d⁻¹)

(c) temperature (K) & res. mass streamf. (Mts⁻¹)

(d) zonal wind (ms⁻¹) & Div (EPF) (ms⁻¹d⁻¹)
Simulated annual-mean radiation budget at the top of the atmosphere

The KMCM is energetically consistent because each conversion preserves energy. Errors are due to aliasing only.

- intensity of the Lorenz energy cycle $\sim 1.9 \text{ Wm}^{-2}$
- latent + sensible surface heat flux $\sim 80 \text{ Wm}^{-2}$
- mean ASR $\sim$ mean OLR $\sim 230 \text{ Wm}^{-2}$
- SR absorbed by the surface / atmosphere $\sim 152 / 78 \text{ Wm}^{-2}$
- mean error in the RTOA $\sim -0.20 \text{ Wm}^{-2}$
- evaporation - condensation $\sim -0.15 \text{ Wm}^{-2}$
- error of the surface energy budget $\sim -0.14 \text{ Wm}^{-2}$
The mean annual cycle is characterized by regular variations with heat storage/release during NH winter/summer along with global cooling/warming, giving rise to a positive/negative RTOA.

Importance of this bottom-up causality for the RTOA in the context of the present stagnation of global warming?
Conclusions

The mesosphere/lower thermosphere (MLT, 50-110 km) is a challenging altitude range to test circulation models with regard to mesoscale dynamics and parameterizations to solve the closure problem.

Current circulation models do not converge with increased spatial resolution, presumably because of insufficient closures.

Strategy of the theory & modeling department at IAP:
Apply high resolution and aim at a physically consistent parameterization in terms of the Dynamical Smagorinsky Model and assuming an inertial regime in the mesoscales that is governed by stratified turbulence.

A mechanistic model is well suited to develop and test new closures. It can also be formulated as a semi-realistic climate model with consistent energetics by virtue of model formulation.

The annual cycle of the radiation and surface budgets may serve as a benchmark for complex climate models.
spectral fluxes of kinetic energy (m²/s²/d) owing to horizontal advection, adiabatic conversion, and energy deposition.

Global kinetic energy spectrum and normalized spectral tendencies (KMCM T240/L190 with realistic gravity-wave effects in the mesosphere) (Becker & Brune, 2014, JAS)