

## Motivation and theoretical background

**Goal** (here exemplified with the help of the Held Suarez test run with the ICON-IAP model):

- understanding and visualization of net mass fluxes in the atmosphere
- generalized nonlinear and threedimensional viewpoint of the residual circulation

**Constituting steps for the generation of transformed equations as seen with local dynamics and wave dynamics:**

### Local dynamics

1) inactive flow? (no inherent assumptions)

$$\vec{v}_{ia} = \frac{\nabla\theta \times \nabla B}{\rho P}$$

B: Bernoulli function  
P: Ertel's potential vorticity (EPV)  
Θ: potential temperature

$$\rho \vec{v}_{ia} P = \nabla\theta \times \nabla B \quad \text{inactive EPV-flux}$$

2) transformation of equations and definition of active wind

$$\vec{v}_a = \vec{v} - \vec{v}_{ia} \quad \vec{v}_a: \text{active wind}, \vec{v}_{ia}: \text{inactive wind}$$

$$\partial_t \vec{v} - \vec{R} = -\vec{\omega}_a \times \vec{v}_a + c_p \Pi_a \nabla\theta \quad \Pi_a := \Pi - \frac{\vec{\omega}_a \cdot \nabla B}{c_p \rho a \vec{\omega}_a \cdot \nabla\theta}$$

$$\partial_t \theta - Q = -\vec{v}_a \cdot \nabla\theta$$

$$\partial_t \rho - \rho \vec{v}_{ia} \cdot \nabla \ln P = -\nabla \cdot (\rho \vec{v}_a)$$

$$\partial_t (\rho P) = -\nabla \cdot (\rho \vec{v}_a P - \vec{\omega}_a Q + \nabla\theta \times \vec{R})$$

3) time/zonal mean of EPV-flux equation

$$-\nabla\theta \times (\vec{\omega}_a \times \vec{v}_a) + \vec{\omega}_a (\vec{v}_a \cdot \nabla\theta) = \rho \vec{v}_a P$$

$$\nabla\theta \times (\partial_t \vec{v} - \vec{R}) - \vec{\omega}_a (\partial_t \theta - Q) = \rho \vec{v}_a P + \rho \vec{v}_a P'$$

$$\rho \vec{v}_a P' \quad \text{imitates the role of the Eliassen Palm flux divergence}$$

### Wave dynamics

1) time/zonal mean of u- and θ- equations

$$-v^s = \bar{v} = \frac{1}{\bar{\rho}} \frac{\partial (\overline{\rho v}) \theta'}{\partial z \partial \theta / \partial z}$$

$$-w^s = \bar{w} = -\frac{1}{\bar{\rho} \alpha \cos \varphi} \frac{\partial \overline{\cos \varphi (\rho v) \theta'}}{\partial \theta / \partial z}$$

3) transformation of u- and θ- equations (Transformed Eulerian Mean)

$$\partial_t \bar{u} - \bar{R}_x = v^*(f + \zeta) - w^* \partial_z \bar{u} + \rho_r^{-1} \nabla \cdot \overline{EPF}$$

$$\partial_t \bar{\theta} - \bar{Q} = -w^* \partial_z \bar{\theta} \quad \rho_r^{-1} \nabla \cdot \overline{EPF}_{qg} = v^* q'$$

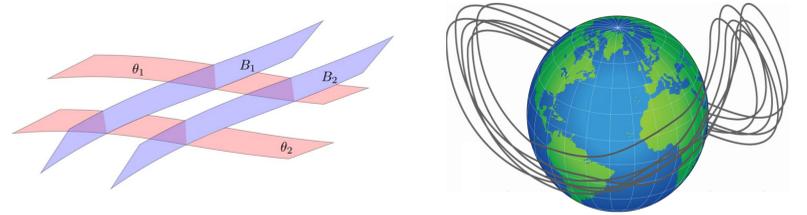
$$0 = -\rho_r \partial_y v^* + \partial_z (\rho_r w^*) \quad q: \text{quasigeostrophic PV}$$

$$v^* = \bar{v} - \bar{v} \quad \text{and} \quad w^* = \bar{w} - \bar{w}$$

### Advantages of local viewpoint and local transformation:

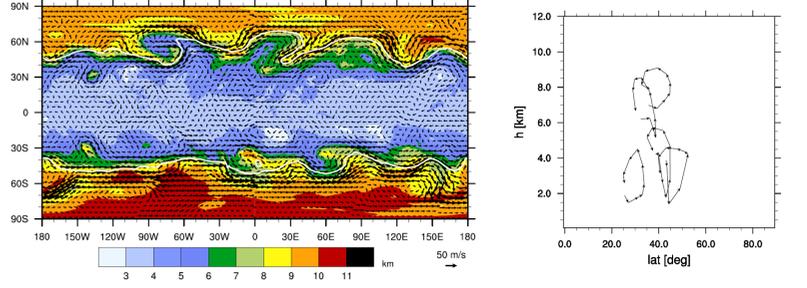
- The central role of the EPV-flux underlines the concerted action of momentum and heat fluxes. This aspect is not so obvious in the TEM equations. There, the (qg) PV-flux is counted within the momentum equation even though it has aspects of both: hydrodynamics and thermodynamics.
- The contribution of Ekman dynamics in the atmospheric boundary layer becomes obvious. This is impossible to be taken into account with the concept of the residual circulation, because the Stokes drift is unable to see the Earth's surface.
- The contribution of the diabatic processes constitutes a net vertical mass flux.
- The renouncement of the wave dynamical viewpoint allows for averaging over nonperiodic domains or in time.

## Inactive wind as dominating wind in the atmosphere

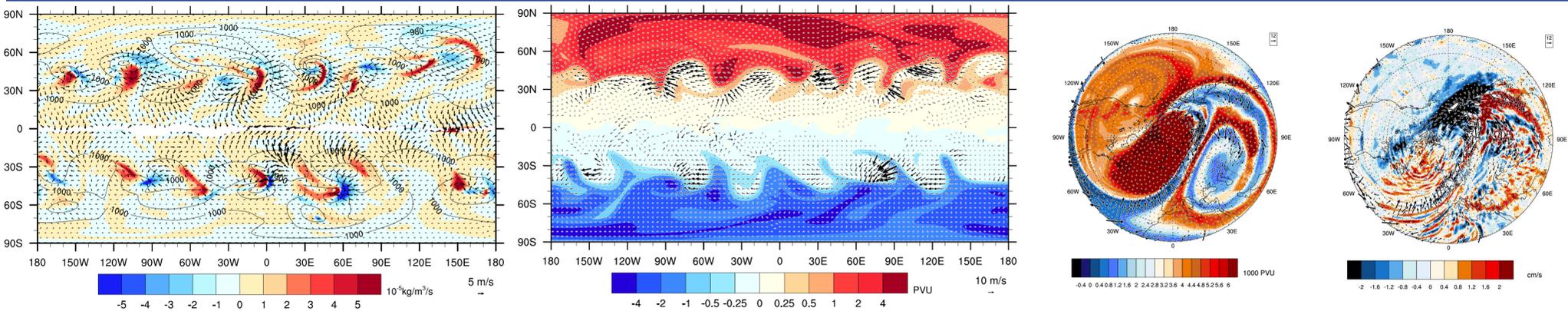


The inactive wind blows along closed streamlines – the intersection lines of B- und θ-surfaces (upper left). The upper right sketch exaggerates the B-isolines on an isentropic surface as is given for a real case in the lower right panel (B=314 kJ/kg, θ=315 K). Actual wind vectors follow the streamlines closely. Colors mark the height of the isentrope. The lower right sketch shows longitudinally consecutive intersection points of B and θ in the meridional plane on the northern hemisphere. As expected for Rossby waves, one encounters elliptically shaped paths, where the lowermost point is located further north than the uppermost point.

The dynamical state index (DSI) of Weber und Névir (2008) is equal to the advection of Ertel's enstrophy by the inactive wind,  $DSI := \vec{v}_{ia} \cdot \nabla P^2 / 2$ . If the DSI is deviating from zero, the atmosphere is in an unbalanced state. Extreme weather events may occur. A weighted DSI is present in the transformed continuity equation as a source term:  $DSI^* := \rho \vec{v}_{ia} \cdot \nabla \ln P$ . A deviation of the DSI from zero indicates a transition between active and inactive mass flux. The residual mass flux of the TEM equation is however non-divergent. This means that ideal wave parts (represented by the Stokes drift) and real wave parts (represented by the residual circulation) do not interfere with each other.



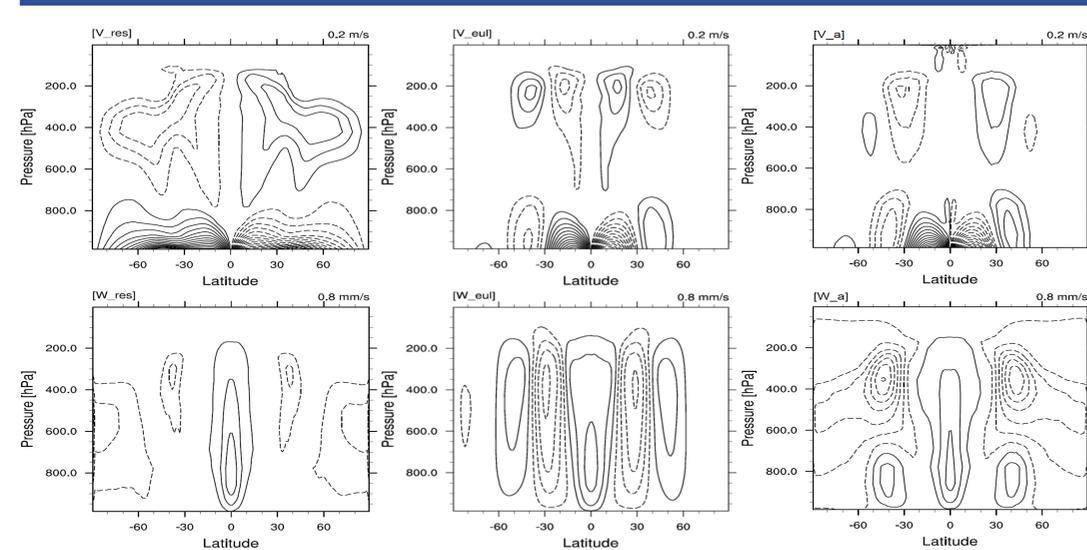
## Local active wind



The left panel shows the horizontal active wind vectors in the atmospheric boundary layer together with the surface pressure (contours) and  $DSI^*$  (colors). If air is sinking in the cold part of the cyclone, the flow leaves the inactive path and is influenced by friction. This means a sink for the inactive flow part and a source for the active flow part. The weighted DSI is positive, then. The contrary is true for the warm conveyor belt.

Filamentation of vorticity streamers is an indication of breaking Rossby waves. Quite frequently, poleward active winds are present in the region of low absolute values of EPV (middle panel). In the region of high absolute EPV values, active winds are weak and point equatorward. This is valid for synoptic and planetary waves in the upper troposphere/lower stratosphere (middle panels). The active vertical wind in the stratosphere (right panel) displays the signature of gravity waves and slow sinking.

## Time and zonal mean of the active winds compared to residual and Eulerian mean winds



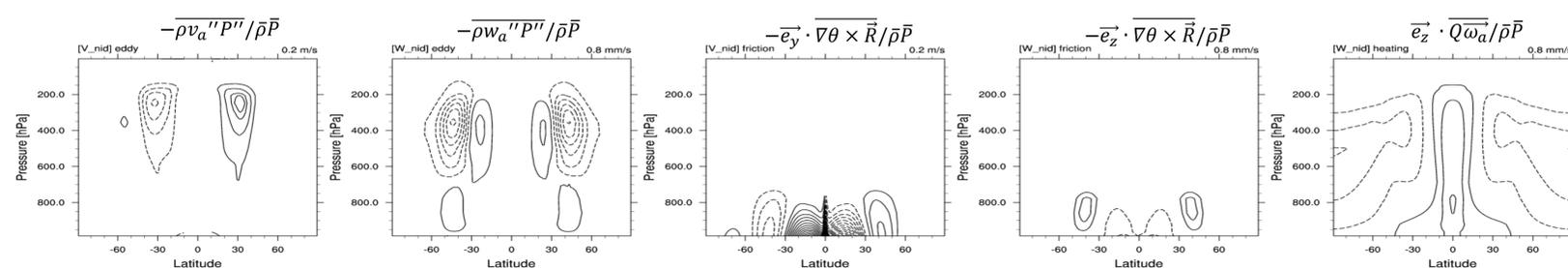
**Left:** residual wind  
**Center:** Eulerian mean wind  
**Right:** mean active wind

**Top:** meridional wind  
**Bottom:** vertical wind

Contributions of the eddies (transience), friction and diabatic effects to the mean active winds are displayed in the lower row.

Contrary to the residual and Eulerian mean winds, the mean actual wind is not non-divergent in the meridional plane. The total circulation is taking place partly with the active and partly with the inactive flow.

Within the zone of the breaking baroclinic waves (ca. 300 hPa, 30°N/S) both, the residual and the mean active wind, are pointing poleward. Diabatic contributions are associated with rising in the tropics and sinking near the poles. Differences in the vertical motions are mainly due to the approximations done when computing the Stokes drift. There, only Earth vorticity  $f$  and not absolute vorticity  $\vec{\omega}_a$  is taken into account. Ekman dynamics as represented by sinking in subtropical highs and raising in mid-latitude lows are visible for the mean active wind. Ekman dynamics may not be represented by the residual wind.



## References

- Gassmann, A. 2014: Deviations from a general nonlinear wind balance: Local and zonal-mean perspectives. Meteorol. Z. 23 (5), 467-481.
- Weber, T. and Névir, P. 2008: Storm tracks and cyclonic development using the theoretical concept of the Dynamic State Index DSI. Tellus 60 (A), 1-10.