

Zonal index persistence, eddy feedbacks and their sensitivity to friction



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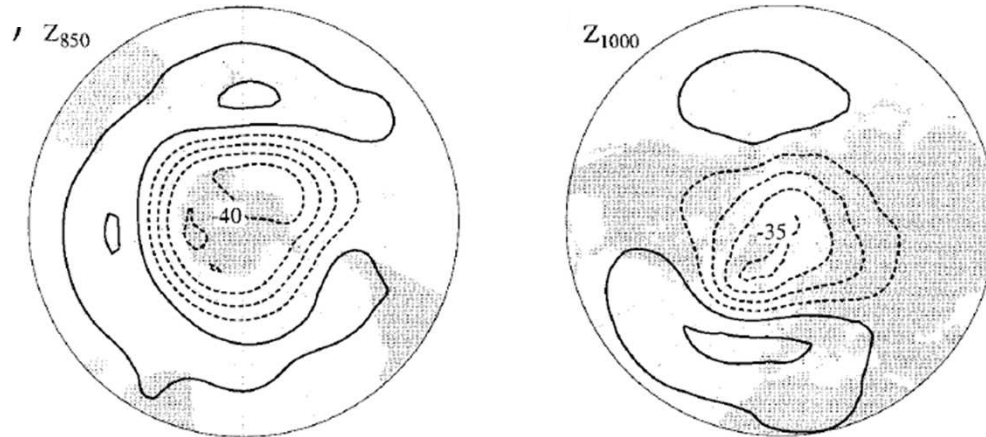
CliSAP workshop
Hamburg, May 13th 2014

Annular modes

Leading mode of variability of extratropical SLP/height in both hemispheres.

Associated with *a mass exchange between middle and high latitudes.*

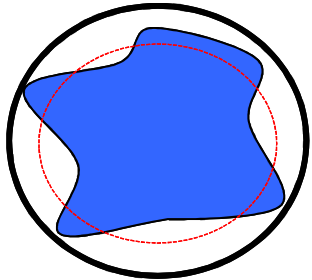
Thompson and Wallace (2000)



Important because:

- Dominate the variance at most time scales
- Pervasive in idealized & comprehensive models
- Preferred mode of response for many forced perturbations

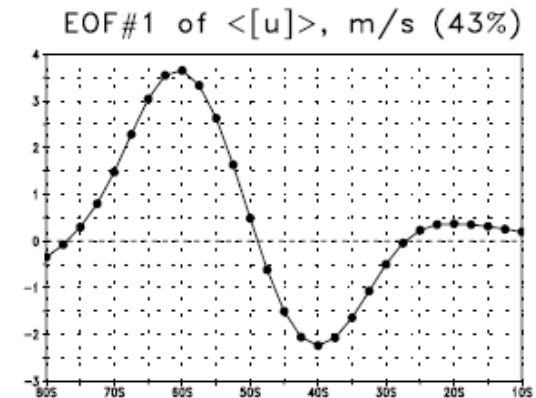
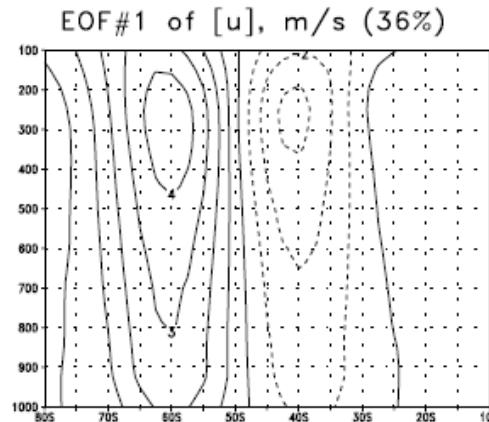
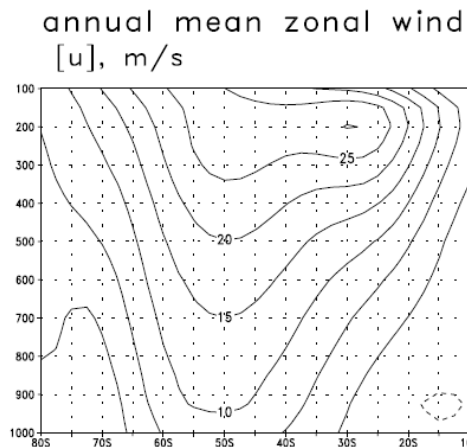
Jet shifts



Annular mode: mass exchange between mid and high latitudes

Polar-front (or eddy-driven) jet : boundary between both regions

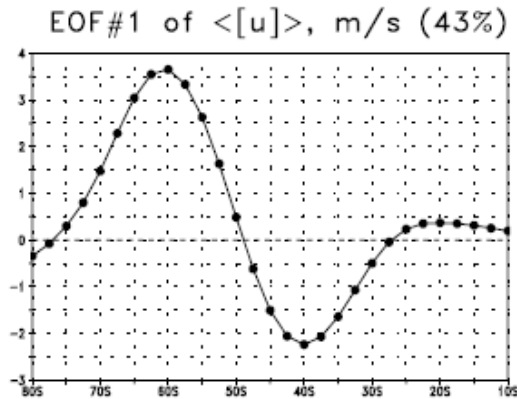
Annular mode \leftrightarrow *shift of the eddy-driven jet*



Lorenz and Hartmann (2001)

The zonal index is defined as: $z = \left(\iint [U](y, z) dz \right) \cdot \overline{EOF}$

The zonal index is a measure of the latitude of the jet



$$z = \left(\iint [U](y, z) dz \right) \cdot \overrightarrow{EOF} \quad \text{is the zonal index}$$

Integrate zonal momentum balance vertically & project on EOF :

$$\frac{\partial z}{\partial t} = m - \frac{z}{\tau_z}$$

τ_z : frictional timescale

$$m = - \left[\frac{1}{\cos^2 \varphi} \frac{\partial}{\partial \varphi} \left(\int [u'v'] \cos^2 \varphi dz \right) \right] \cdot \overrightarrow{EOF} \quad \text{is the eddy momentum forcing}$$

All the dynamics is encapsulated in just one term: the eddy forcing m !

What is the effect of m on the persistence of jet shifts?

Look at zonal index autocovariance/autocorrelation: $C_{zz} = \langle z'(t), z'(t + \tau) \rangle$

If m were white, z anomalies would decay with the frictional scale τ_z : $C_{zz} = \sigma_z^2 e^{-|\tau|/\tau_z}$

But things are different when m has memory!

$$\frac{\partial z}{\partial t} = m - \frac{z}{\tau_z} \quad \longrightarrow \quad \frac{\partial \log C_{zz}}{\partial \tau} = \frac{C_{mz}}{C_{zz}} - \frac{1}{\tau_z} \quad C_{mz} = \langle z'(t), m'(t + \tau) \rangle$$

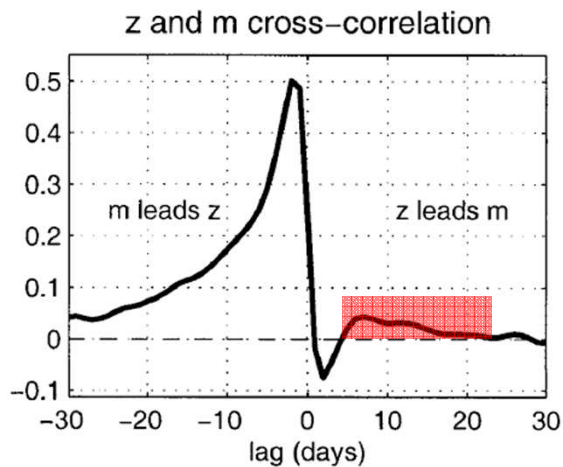
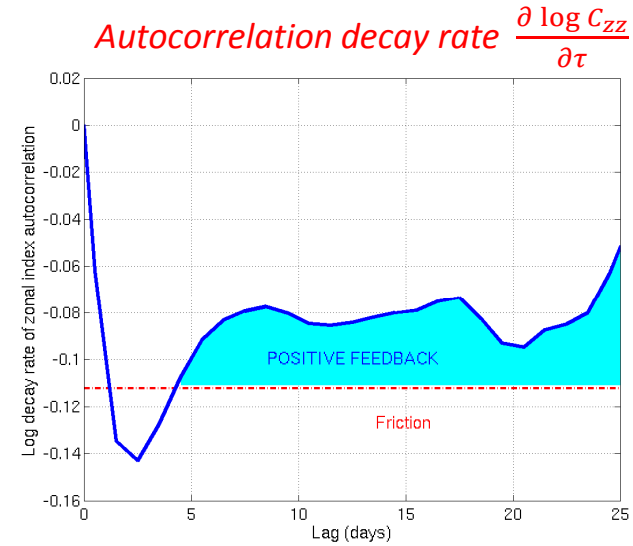
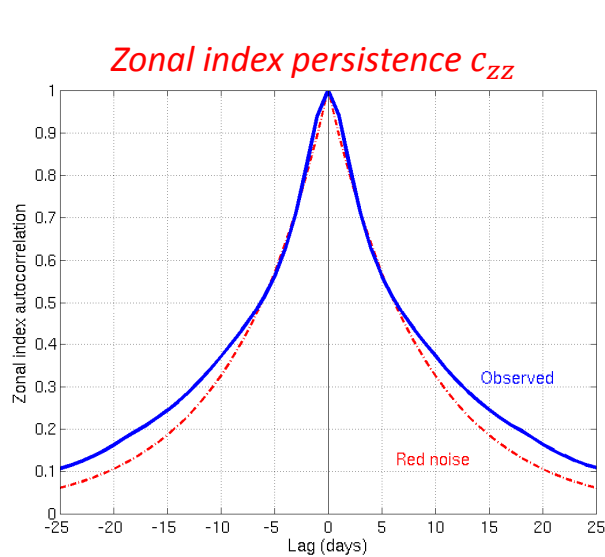
eddy memory effect

$$\frac{\partial \log C_{ZZ}}{\partial \tau} = \frac{C_{mZ}}{C_{ZZ}} - \frac{1}{\tau_Z}$$

$$C_{ZZ} = \langle z'(t), z'(t + \tau) \rangle$$

$$C_{mZ} = \langle z'(t), m'(t + \tau) \rangle$$

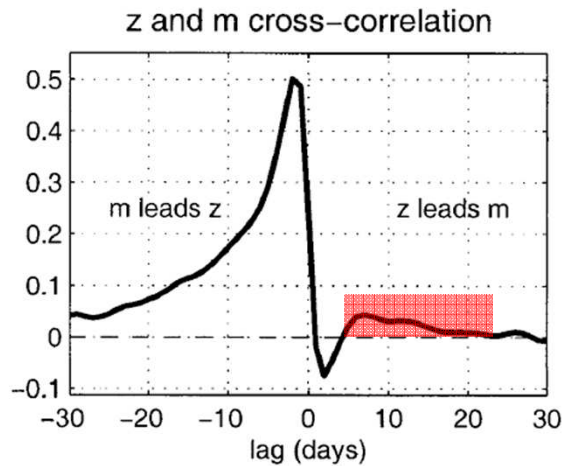
In the atmosphere, eddy memory increases long-lag predictability of jet shifts



Enhanced persistence with positive c_{mZ} correlation
 Physically, **anomalous eddy forcing reinforces z anomalies**

Lorenz and Hartmann (2001)

Linear feedback model



Lorenz and Hartmann (2001)

Enhanced persistence with positive c_{mz} correlation

$$m = \tilde{m} + bz$$

'Random' eddy forcing, independent of z

Feedback coefficient $b > 0$

1. The eddies are organized by the anomalous mean flow (not just random)
2. Coupling is such that the eddy momentum flux reinforces the mean flow anomaly

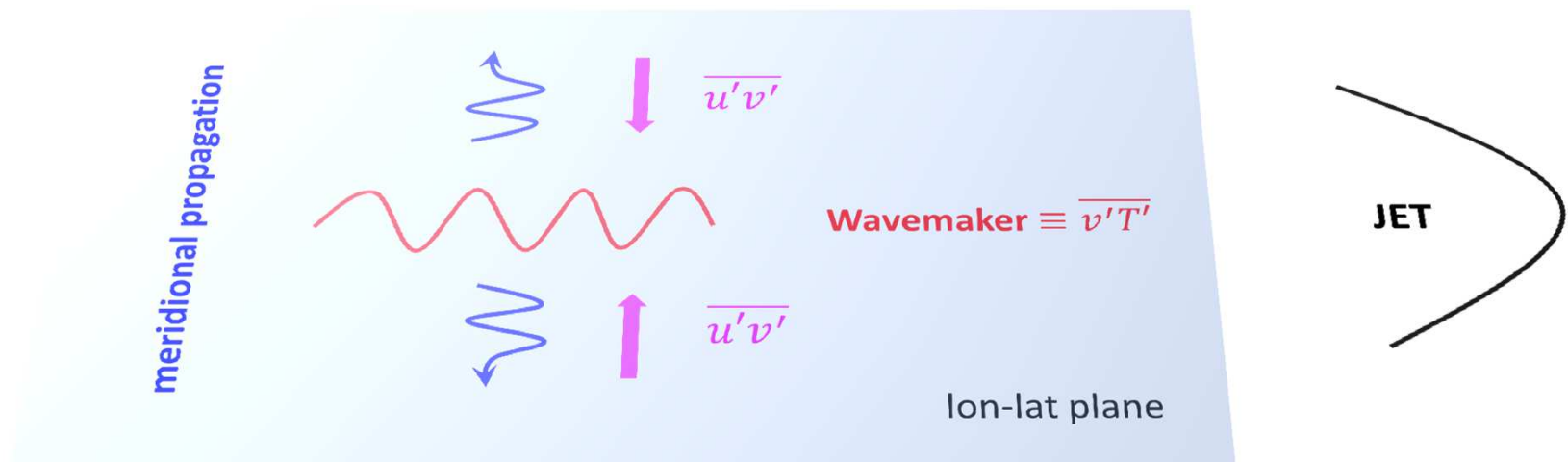
$$\frac{\partial z}{\partial t} = m - \frac{z}{\tau_z} \approx \tilde{m} - (\tau_z^{-1} - b)z \quad \text{equivalent to a reduced friction} \Rightarrow \left\{ \begin{array}{l} \text{more variance} \\ \text{more persistent} \\ \text{preferred response} \end{array} \right.$$

How can we understand this positive feedback?

A theory for the eddy momentum forcing

Conceptual model for the (climatological) eddy momentum flux

- Baroclinic instability generates Rossby waves at upper levels ($\overline{v'T'}$ is a wavemaker)
- Rossby waves propagate away from the generation region at upper levels
- Waves transport momentum in a direction opposite to their propagation
- This results in a westerly acceleration at the generation region.

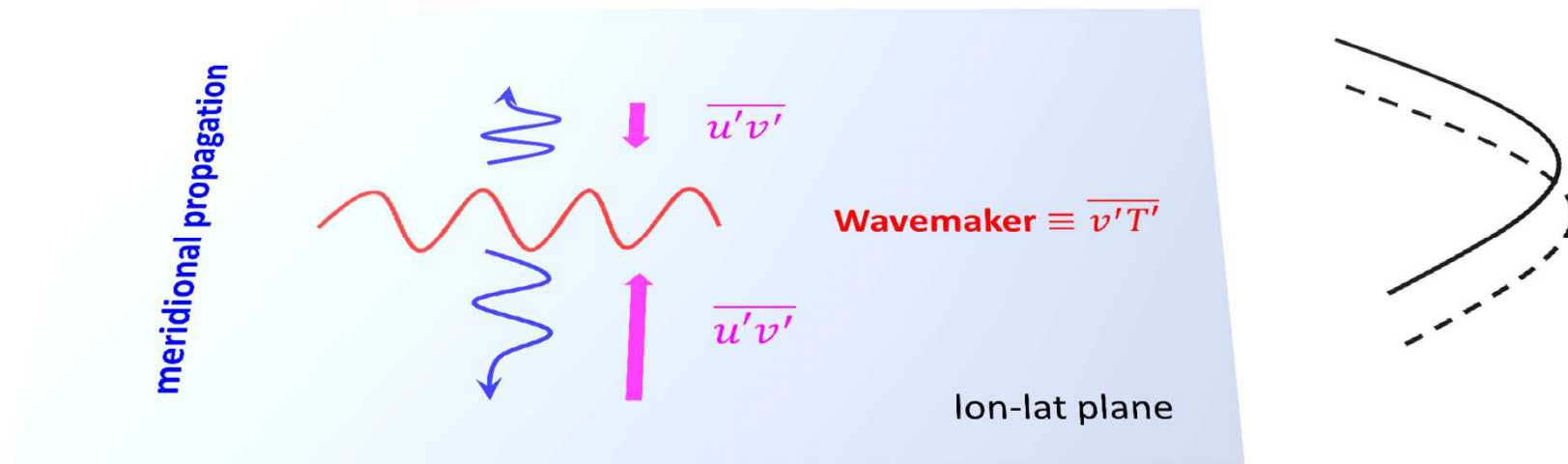


Vallis (2006)

Eddy feedback theories

How can we understand the anomalous eddy momentum fluxes with an anomalous jet?

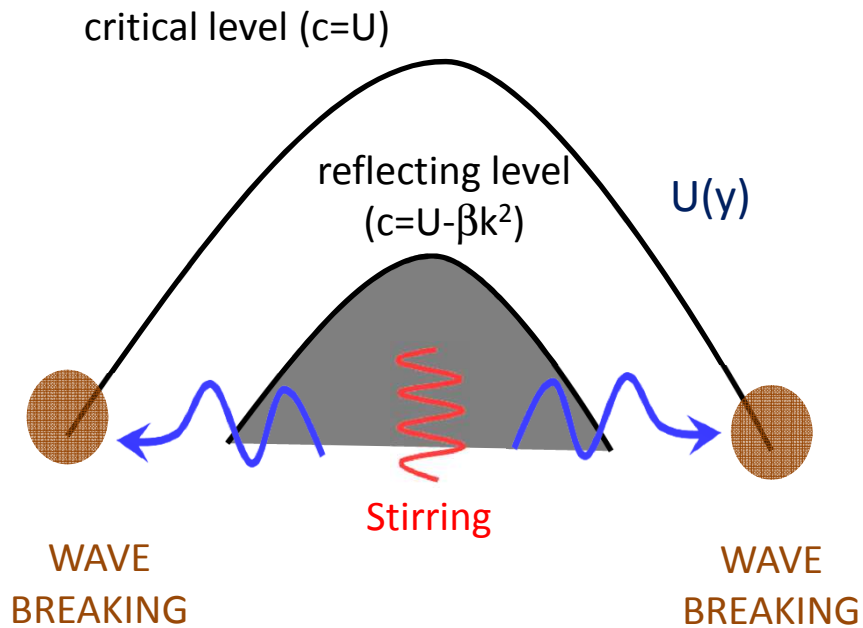
- **Barotropic theories:** changes in the wavemaker (eddy heat flux) are not important. The anomalous eddy momentum fluxes are due to anomalous propagation at upper levels under the anomalous jet (which affects refraction index, critical & reflecting levels,...)



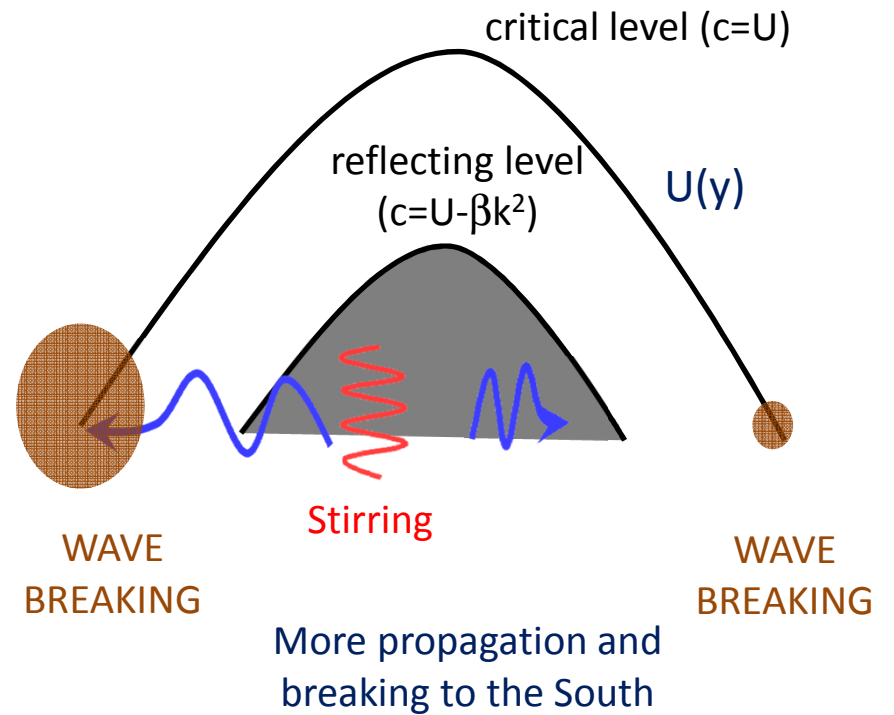
More southward propagation \Rightarrow stronger northward momentum flux \Rightarrow jet pushed further northward

Example (Lorenz 2014)

Centered jet



Jet shifted northward



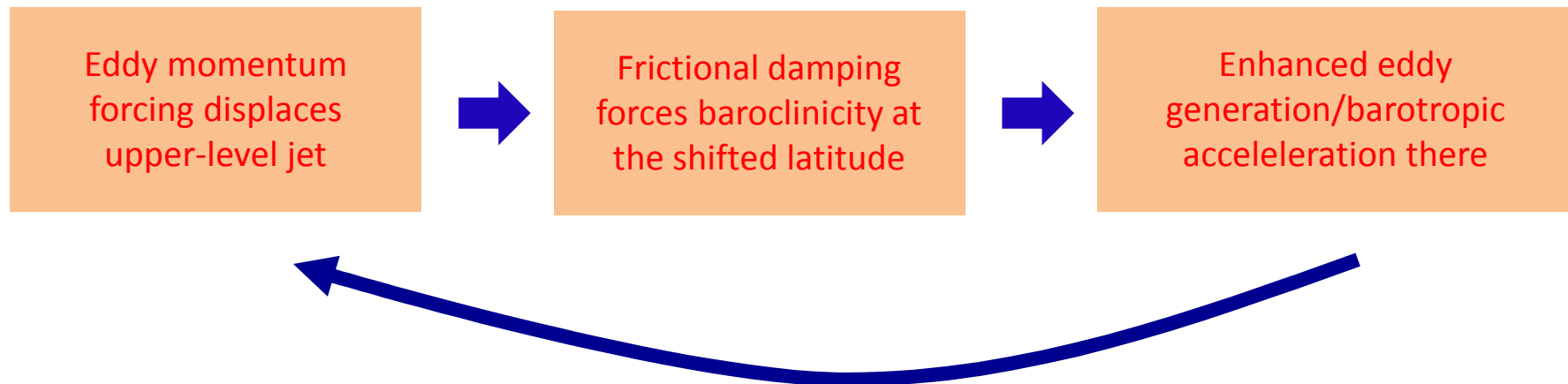
Stirred barotropic models can produce a feedback (though it may be too weak)

We'll focus on baroclinic mechanisms

Eddy feedback theories

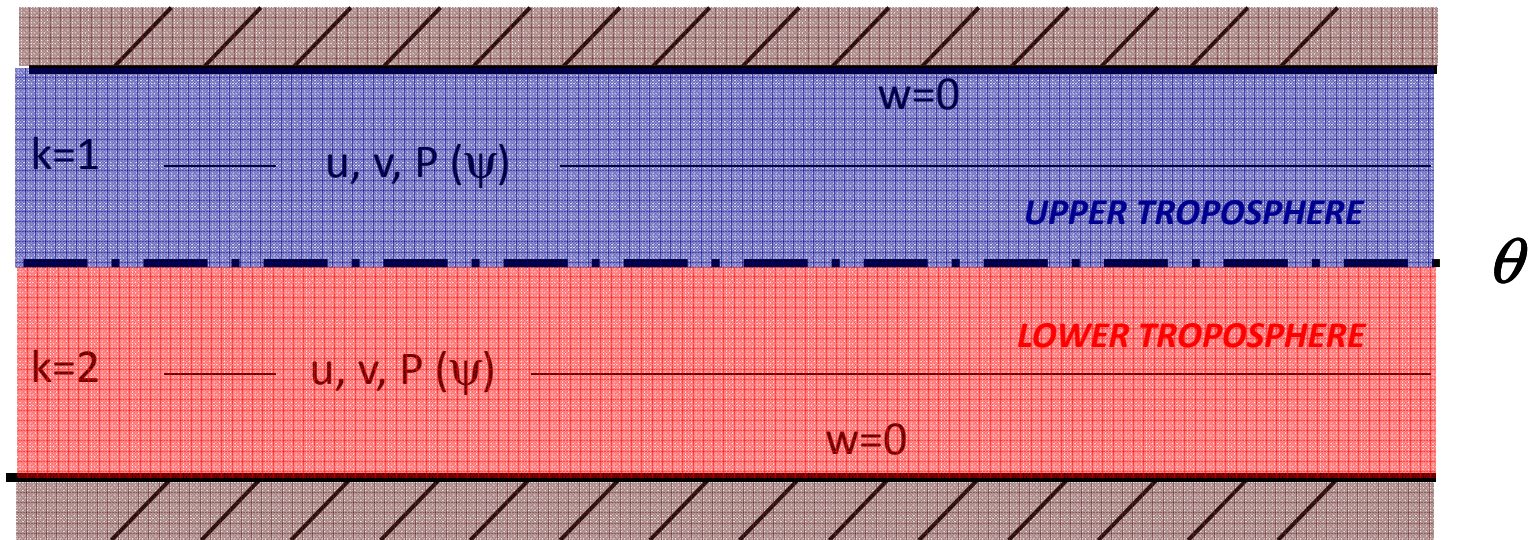
How can we understand the anomalous eddy momentum fluxes with an anomalous jet?

- **Barotropic theories:** changes in the wavemaker (eddy heat flux) are not important. The anomalous eddy momentum fluxes are due to anomalous propagation at upper levels under the anomalous jet (which affects refraction index, critical & reflecting levels,...)
- **Baroclinic theories:** coupled barotropic-baroclinic variability. The anomalous eddy momentum fluxes change the baroclinicity and with it the wavemaker (eddy heat flux)



Robinson (2000, 2006)

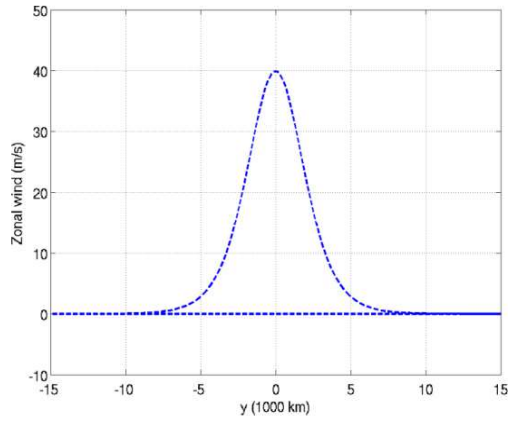
Two-layer QG model



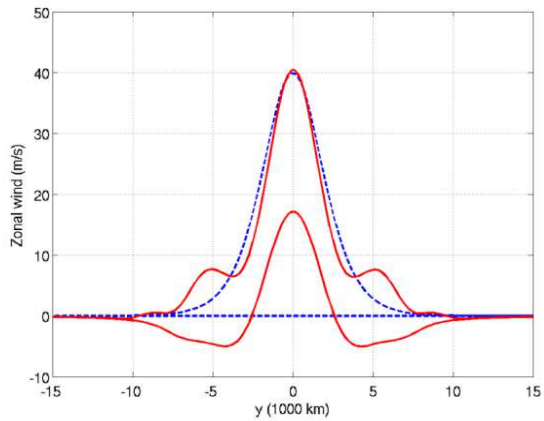
$$\frac{\partial q_1}{\partial t} + J(\psi_1, q_1) = \frac{1}{\tau_D} \frac{\psi_1 - \psi_2 - \psi_R}{\lambda^2} \quad \frac{\partial q_2}{\partial t} + J(\psi_2, q_2) = -\frac{1}{\tau_D} \frac{\psi_1 - \psi_2 - \psi_R}{\lambda^2} - \frac{1}{\tau_F} \nabla^2 \psi_2$$

- A single baroclinic mode, beta channel.
- **Forced by thermal relaxation to an unstable jet.** Damped by Rayleigh friction
- ***This is the simplest model that can produce a baroclinic feedback***

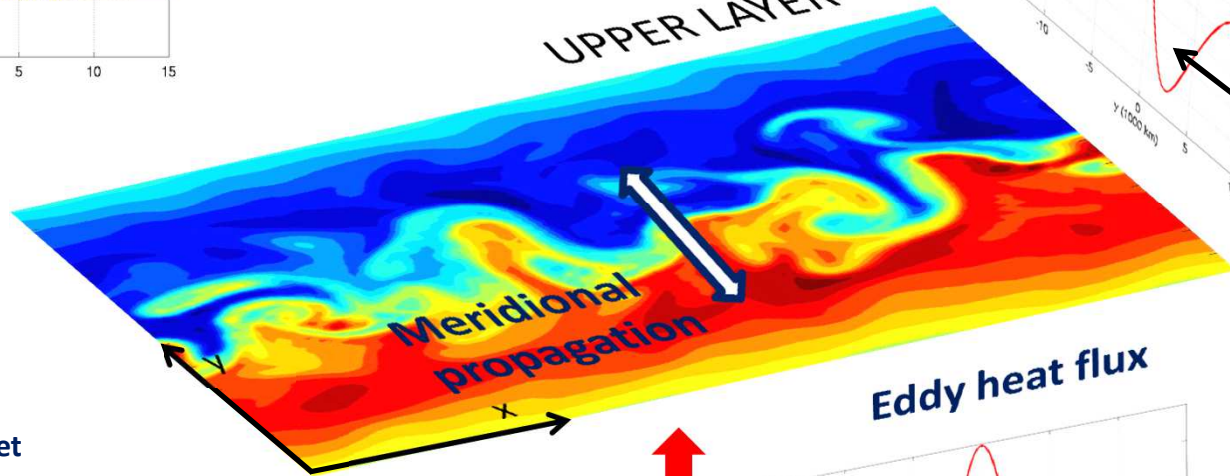
Radiative equilibrium jet



Climatological jet



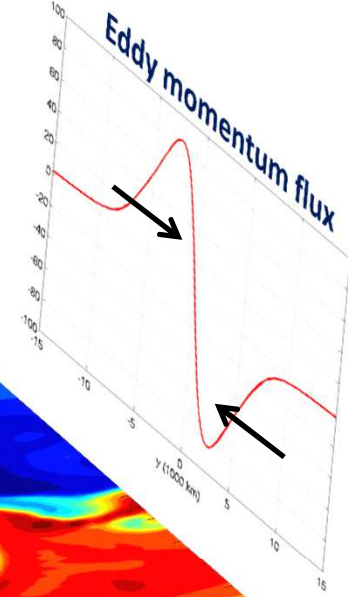
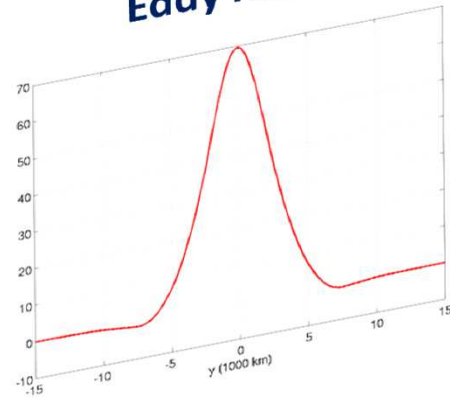
UPPER LAYER



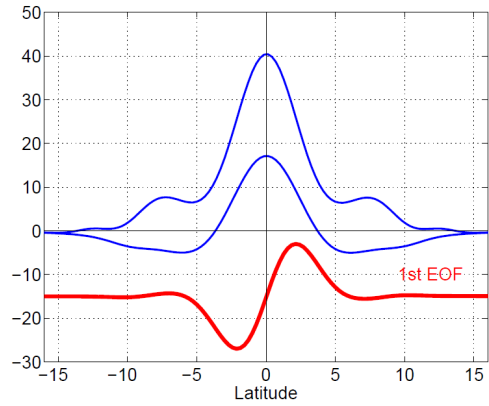
**Eddy heat flux:
wavemaker**



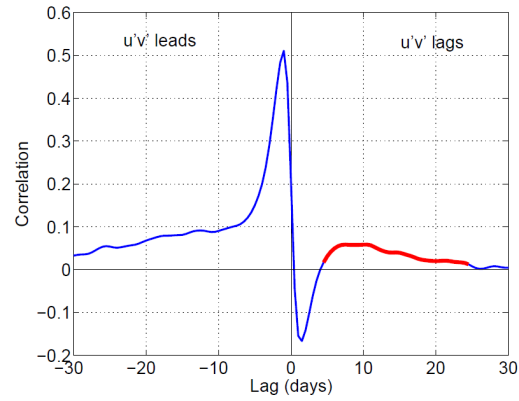
Eddy heat flux



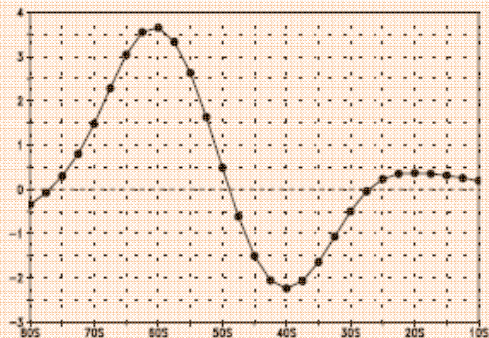
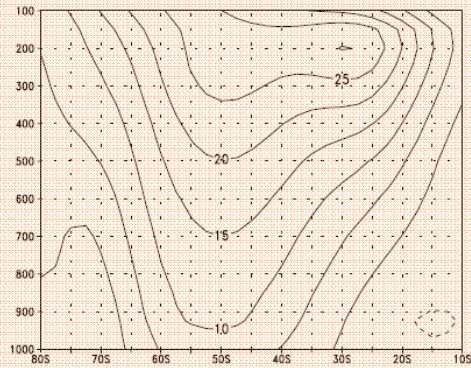
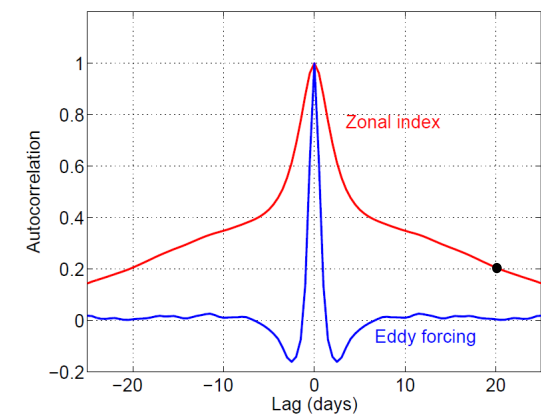
Mean zonal wind and 1st EOF



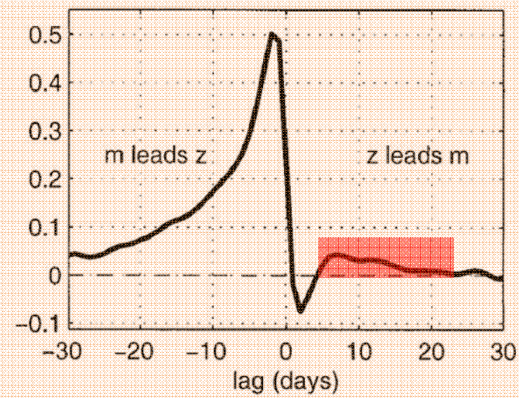
$\langle m, z \rangle$ lagged correlation



z autocorrelation



z and m cross-correlation



c) z auto-correlation

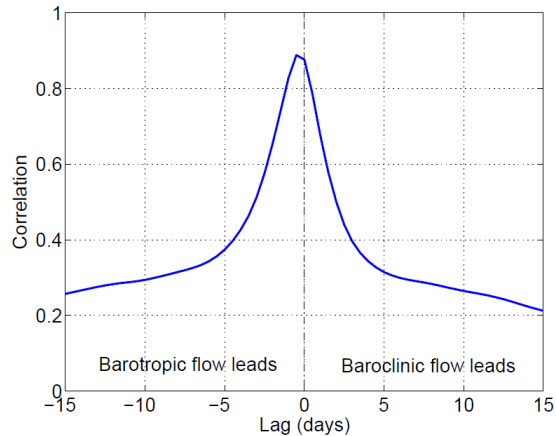


Lorenz and Hartmann (2001)

Coupled barotropic-baroclinic variability

Is the internal variability of the model consistent with the baroclinic feedback mechanism?

Correlation zonal-index/baroclinicity



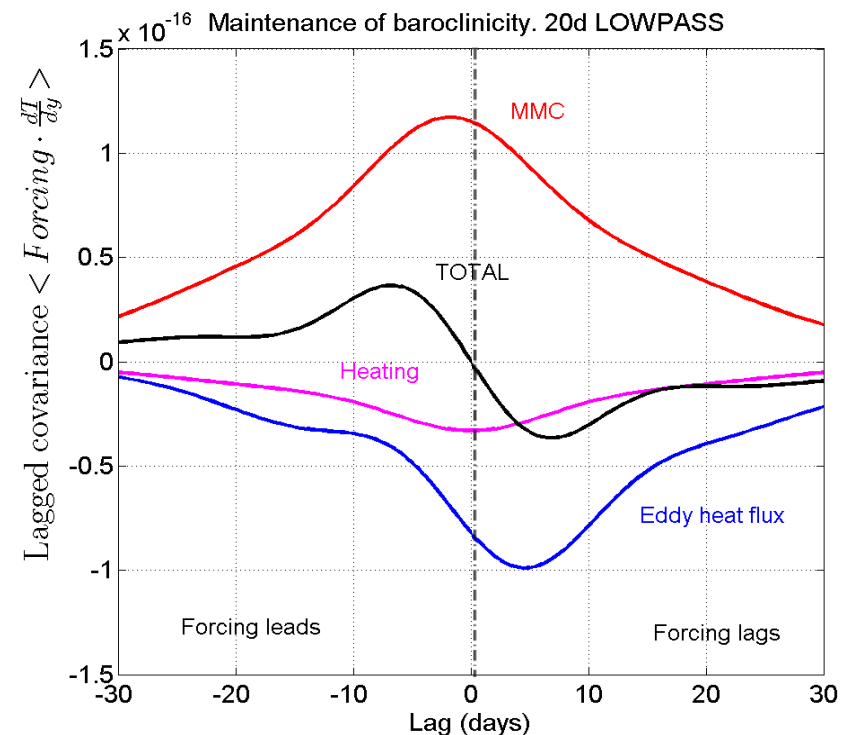
- Leading mode of baroclinicity variability is a shift.
- Strongly correlated with barotropic shift (zonal index)

Driving of baroclinicity shift (low-freq)

$$\frac{\partial \text{Bar}}{\partial t} = F\{\overline{v'\theta'}\} + F\{\text{MMC}\} + F\{\text{Heat}\}$$

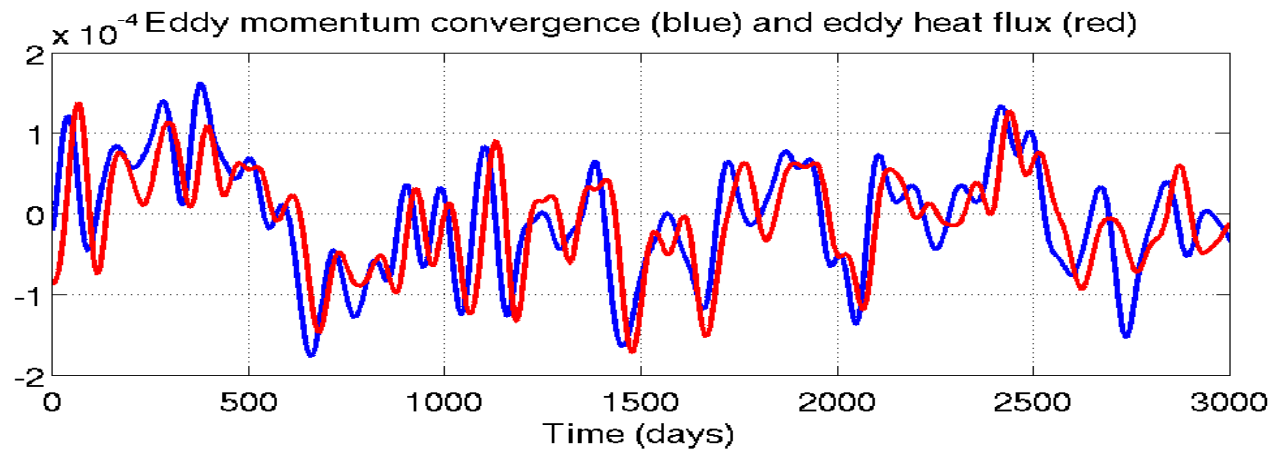
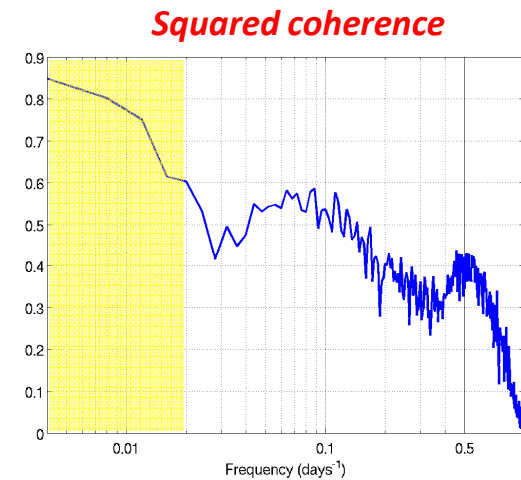
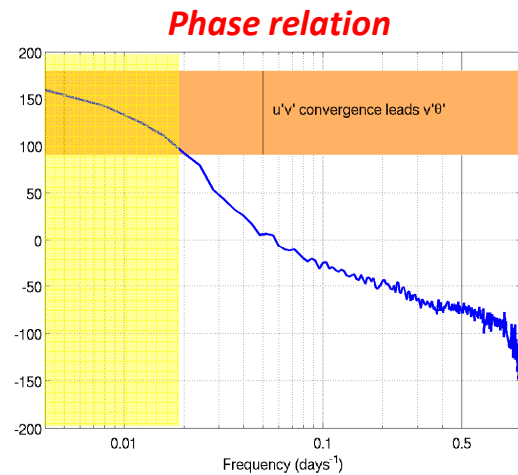
Regression shows shift is driven by mean meridional circulation (MMC), damped by the eddy heat flux and diabatic heating

Zurita-Gotor et al. (2014)



Eddy momentum and heat fluxes strongly correlated, with eddy momentum fluxes leading

**Complex cross-spectrum
between eddy heat and
momentum fluxes**

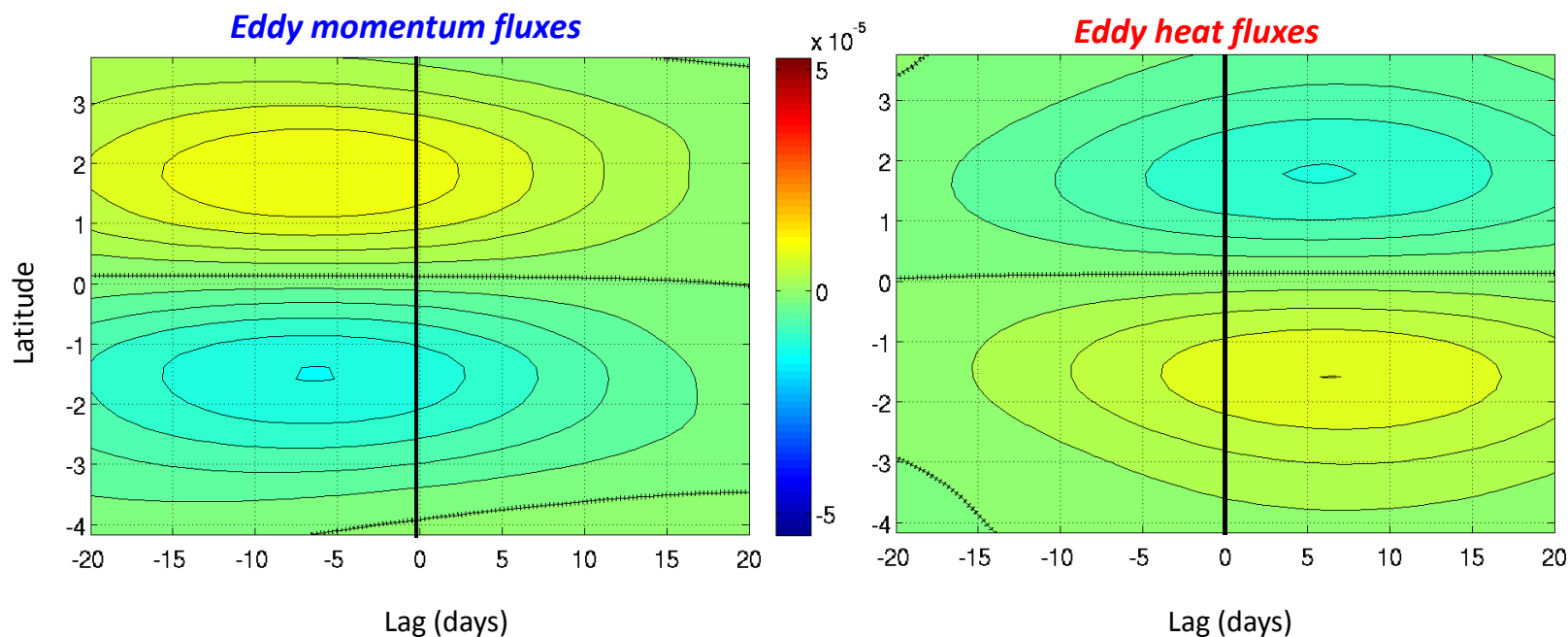


Zurita-Gotor et al. (2014)

Low-frequency internal variability is consistent with the baroclinic feedback mechanism

Regress components of upper-level Eliassen-Palm divergence on *low-frequency* zonal index

$$\nabla \cdot F = -\frac{\partial \overline{u'v'}}{\partial y} - \frac{f_0}{\Delta\Theta} \overline{v'\theta'}$$



Eddy momentum convergence leads zonal index variations, and eddy heat flux lags them

Very different from classical baroclinic lifecycles!

Sensitivity of zonal index persistence to friction

$$\frac{\partial z}{\partial t} = m - \frac{z}{\tau_z} \approx \tilde{m} - (\tau_z^{-1} - b)z$$

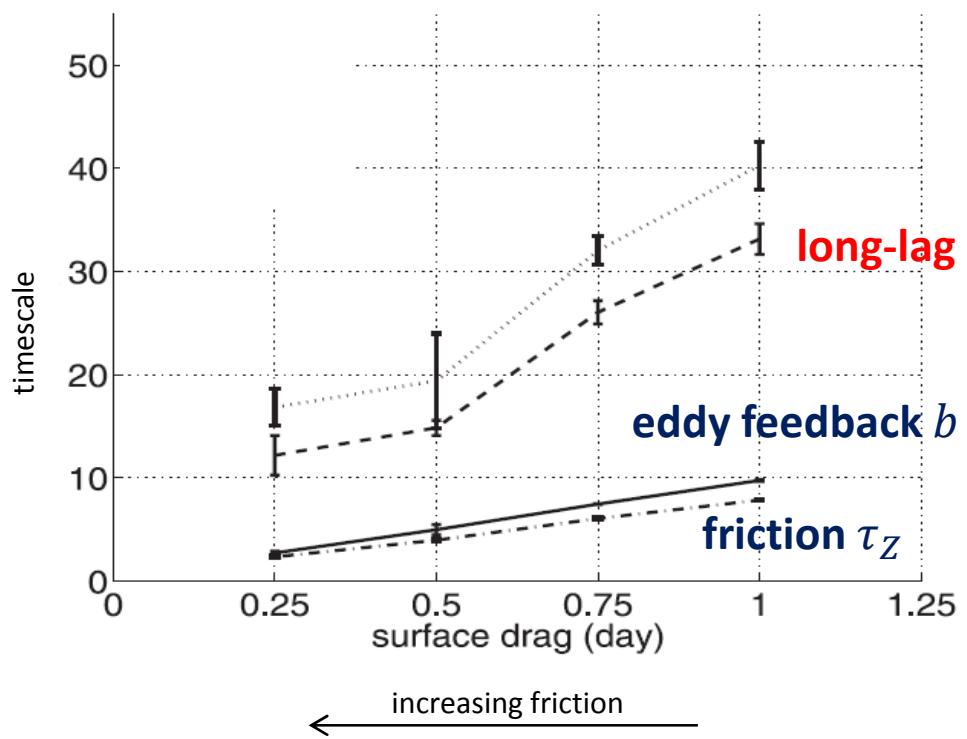
Friction has many confounding effects!

- Friction defines the decorrelation timescale with white m (red noise limit).
- The equilibrium state, and hence the eddy characteristics, are sensitive to friction.
- Friction may change b through the baroclinic feedback mechanism.

Chen & Plumb (2009) studied sensitivity to friction in the Held & Suarez model (primitive equation dry model on the sphere) keeping the basic state fixed and found:

- Eddy feedback increases with friction, almost linearly
- Since they also largely balance, decorrelation timescale is much longer
- Decorrelation timescale is weakly sensitive to friction

$$\frac{\partial z}{\partial t} = \tilde{m} - (\tau_z^{-1} - b)z$$



long-lag zonal index decorrelation $(\tau_z^{-1} - b)^{-1}$

eddy feedback b


friction τ_z

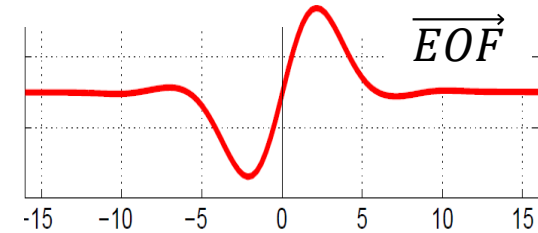
Chen and Plumb (2009)

A smoking gun for the baroclinic feedback mechanism?

We keep the mean state fixed changing friction in the direction of the EOF (shift) alone:

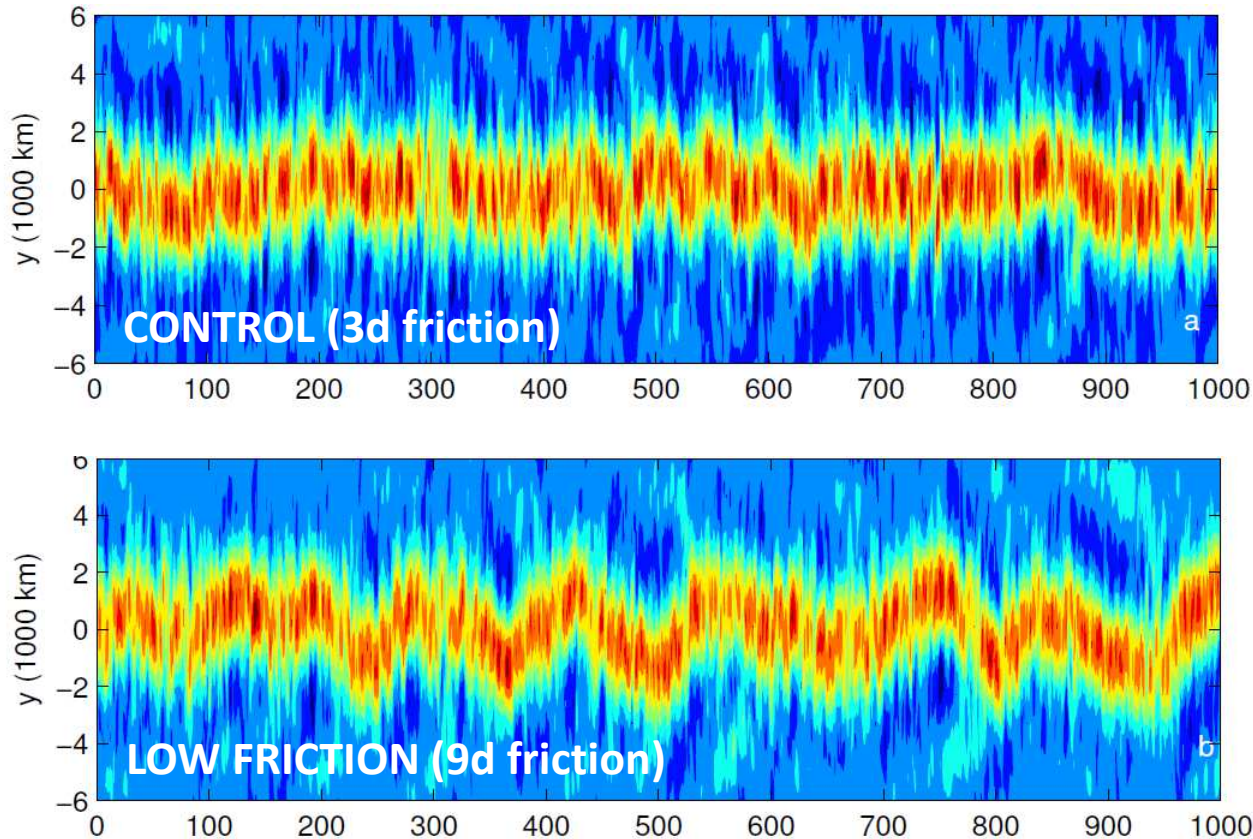
$$\frac{\partial U_2}{\partial t} = \dots - \frac{U_{EOF}}{\tau_{EOF}} - \frac{U_2 - U_{EOF}}{\tau_F}$$


 only this is changed



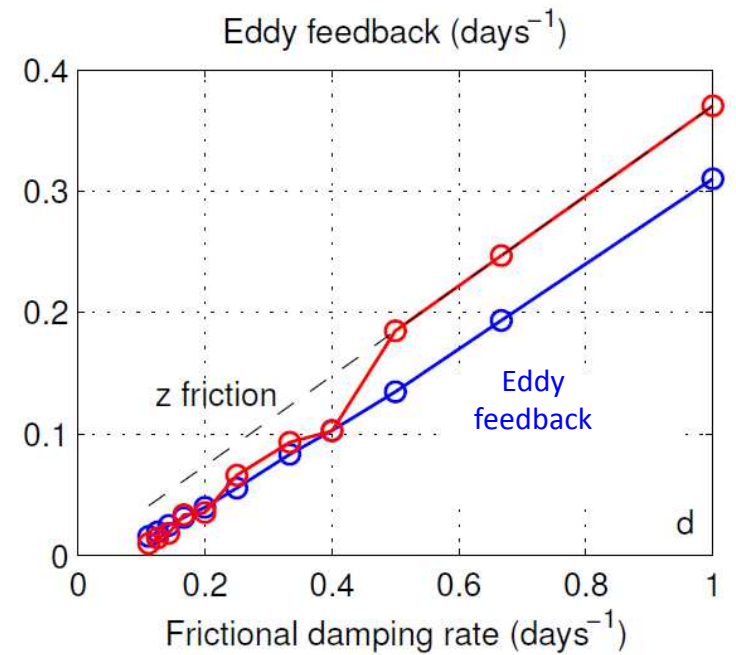
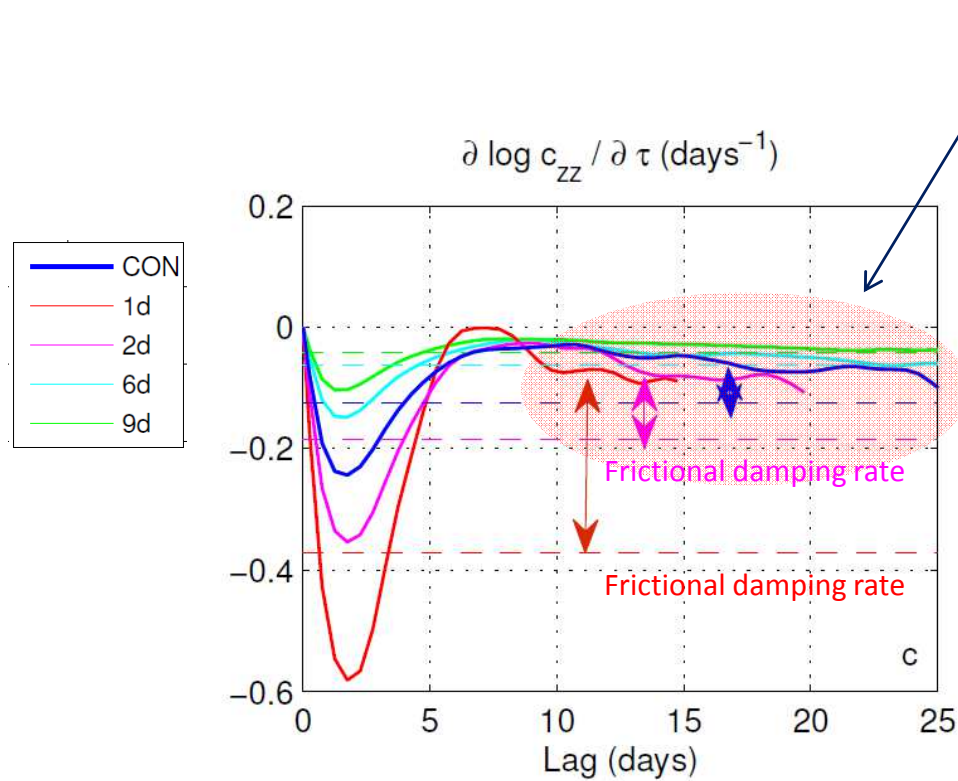
Barotropic wind timeseries

$$U_{EOF} = (U_2 \cdot \overrightarrow{EOF}) \overrightarrow{EOF}$$



Despite their very different variability, both simulations have the same time-mean

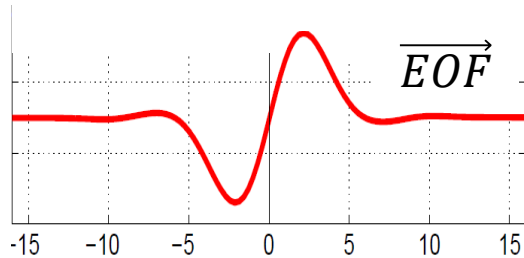
Results are consistent with Chen & Plumb (2009)



Eddy feedback coefficient scales almost linearly with friction

Barotropic vs baroclinic friction

We can also change barotropic and baroclinic friction independently!



$$U_{EOF} = (U_2 \cdot \overrightarrow{EOF}) \overrightarrow{EOF}$$

Upper layer:
$$\frac{\partial U_1}{\partial t} = \dots - \left(\frac{1}{2\tau_{BT}} - \frac{1}{2\tau_{BC}} \right) U_{EOF}$$

Lower layer:
$$\frac{\partial U_2}{\partial t} = \dots - \left(\frac{1}{2\tau_{BT}} + \frac{1}{2\tau_{BC}} \right) U_{EOF} - \frac{U_2 - U_{EOF}}{\tau_F}$$

Friction is calculated using lower-layer wind but the torque is split across both layers

Barotropic limit ($\tau_{BC} = \infty$)

$$\frac{\partial U_1}{\partial t} = \dots - \frac{U_{EOF}}{2\tau_{BT}}$$

$$\frac{\partial U_2}{\partial t} = \dots - \frac{U_{EOF}}{2\tau_{BT}} - \frac{U_2 - U_{EOF}}{\tau_F}$$

(torque split equally in both layers, no shear)

Baroclinic limit ($\tau_{BT} = \infty$)

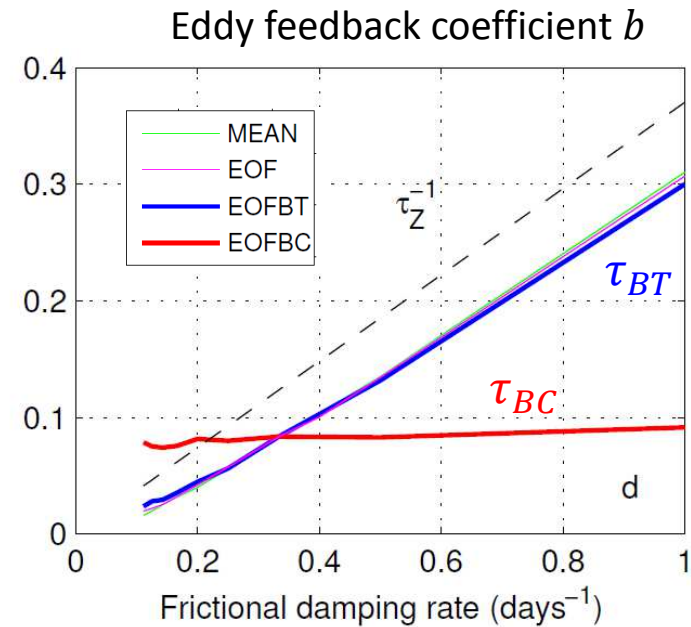
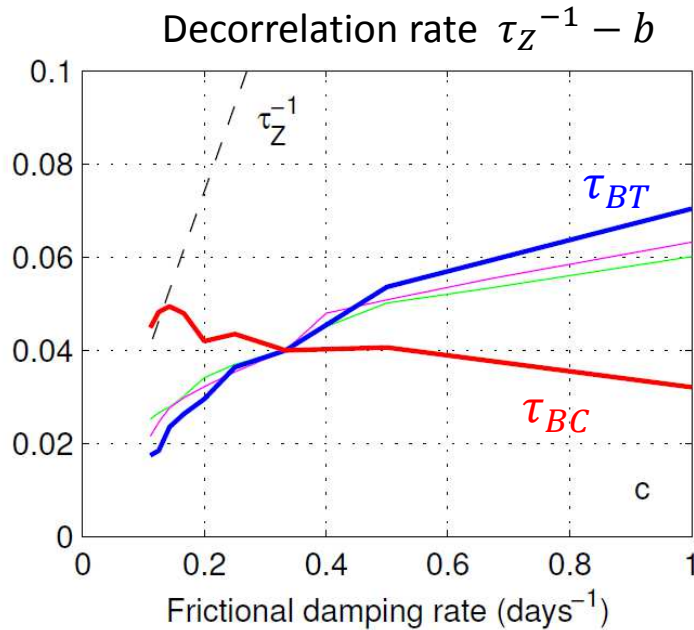
$$\frac{\partial U_1}{\partial t} = \dots \frac{U_{EOF}}{2\tau_{BC}}$$

$$\frac{\partial U_2}{\partial t} = \dots - \frac{U_{EOF}}{2\tau_{BC}} - \frac{U_2 - U_{EOF}}{\tau_F}$$

(opposite-sign torques, no net column force)

With equal values ($\tau_{BT} = \tau_{BC}$) friction has both barotropic and baroclinic effects

$$\frac{\partial z}{\partial t} = \tilde{m} - (\tau_z^{-1} - b)z$$



- Increasing baroclinic friction enhances the eddy feedback but only weakly
- Decorrelation rate changes much less than friction in all cases
- Eddy feedback increases almost linearly with **barotropic** friction!!!

This is not what we expected!!!

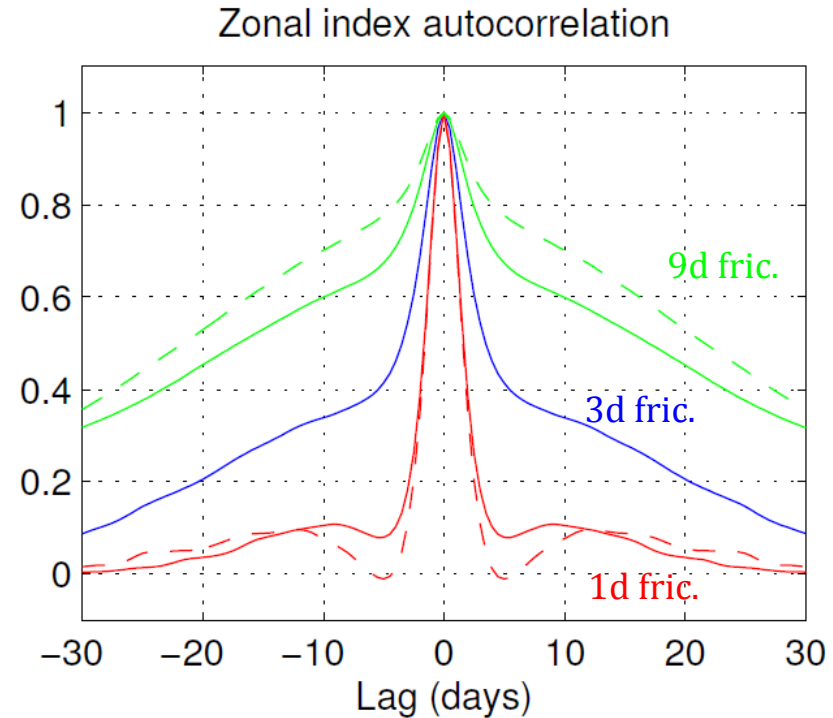
A sanity check...

However, sensitivity to barotropic friction is not due to changes in m .

- Solid lines: zonal index persistence in the model in simulations varying the frictional timescale
- Dashed lines: obtained integrating forward in time the equation:

$$\frac{\partial z}{\partial t} = m - \frac{z}{\tau_z}$$

using [the control \$m\$ timeseries](#)

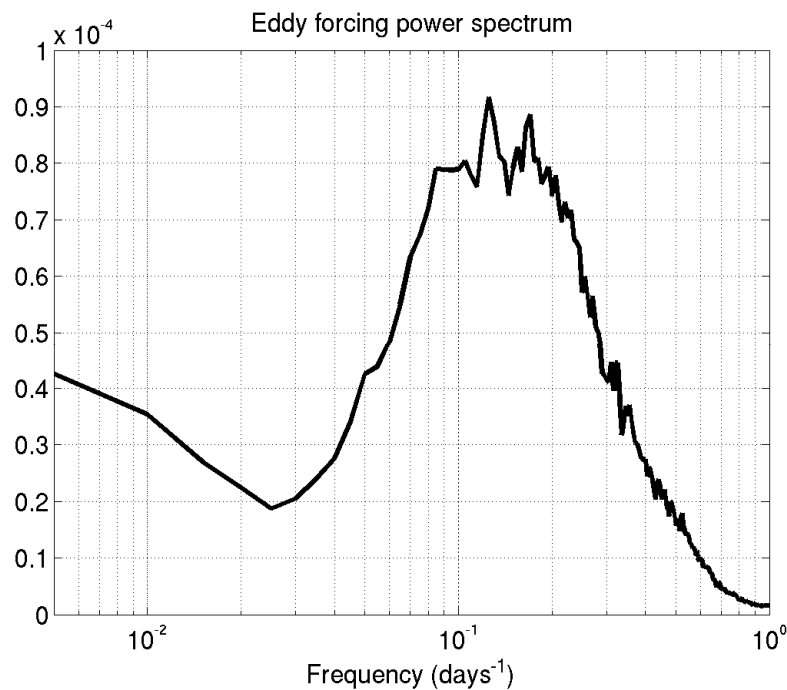


- Changes in the eddy forcing only play a little role for the sensitivity to friction.
- Sensitivity is dominated by the direct damping effect!

Spectral analysis

From Wiener's theorem the power spectrum Z^*Z and the autocovariance C_{ZZ} form a Fourier pair \Rightarrow we can understand changes in C_{ZZ} from changes in the power spectrum

$$\frac{\partial z}{\partial t} = m - \frac{z}{\tau_Z} \quad \longrightarrow \quad i\omega Z = M - \frac{Z}{\tau_Z} \quad \longrightarrow \quad \boxed{Z^*Z(\omega) = \frac{M^*M(\omega)}{\omega^2 + \tau_Z^{-2}}}$$



Eddy forcing spectrum consists of:

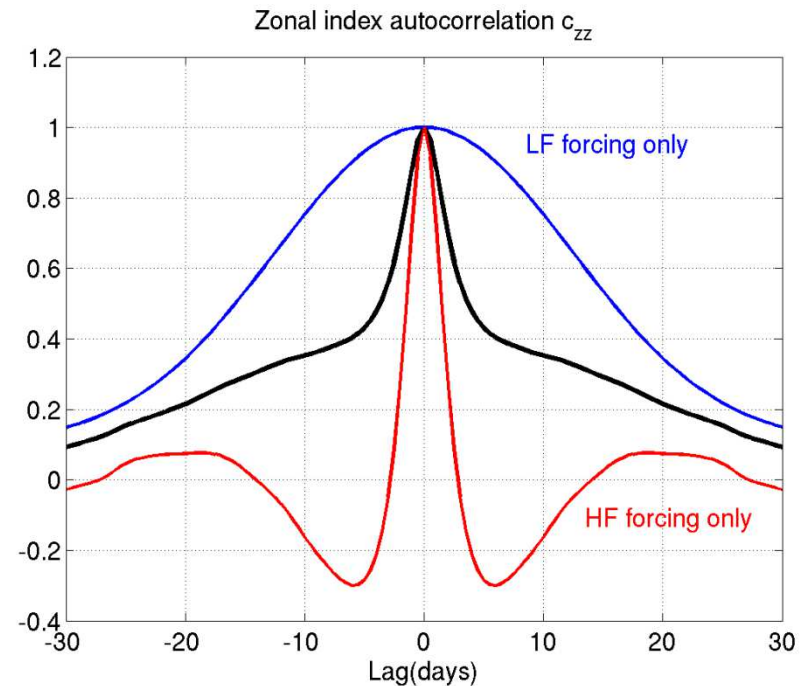
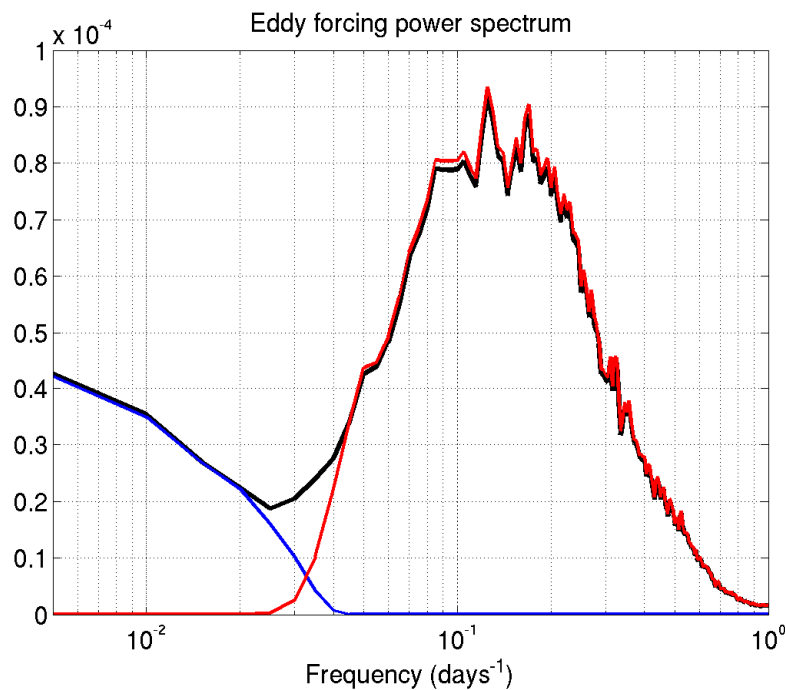
- High frequency peak (synoptic scales)
- Low frequency peak

Spectral analysis

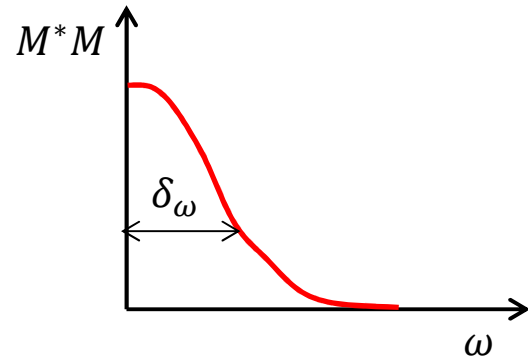
From Wiener's theorem the power spectrum Z^*Z and the autocovariance C_{ZZ} form a Fourier pair \Rightarrow we can understand changes in C_{ZZ} from changes in the power spectrum

$$\frac{\partial z}{\partial t} = m - \frac{z}{\tau_Z} \quad \longrightarrow \quad i\omega Z = M - \frac{Z}{\tau_Z} \quad \longrightarrow$$

$$Z^*Z(\omega) = \frac{M^*M(\omega)}{\omega^2 + \tau_Z^{-2}}$$



Long-lag decay is governed by the **low-frequency peak**



Modeling the *low-frequency* eddy forcing spectrum as a Gaussian peak with spectral width δ_ω :

$$M^*M(\omega) = Ae^{-\omega^2/\delta_\omega^2} \quad Z^*Z(\omega) = \frac{Ae^{-\omega^2/\delta_\omega^2}}{\omega^2 + \tau_Z^{-2}}$$

There are two limits:

- **White forcing limit** (broad peak): $\delta_\omega \gg \tau_Z^{-1}$

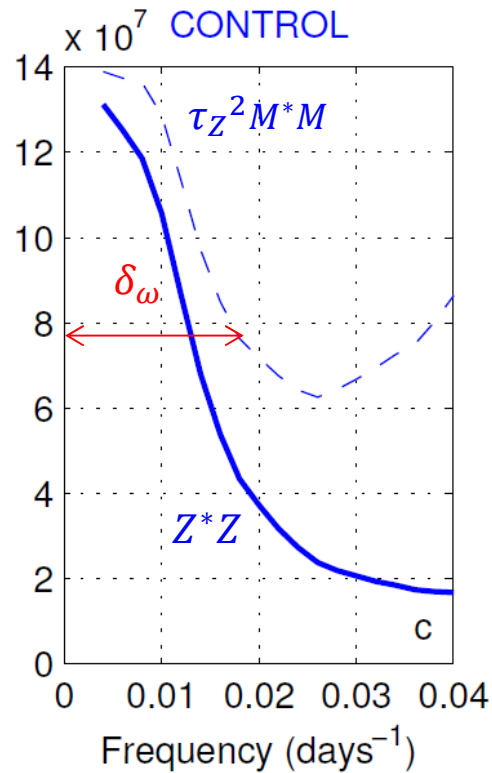
$$Z^*Z(\omega) \approx \frac{A}{\omega^2 + \tau_Z^{-2}} \quad \Rightarrow \quad C_{ZZ}(\tau) = Ae^{-|\tau|/\tau_Z} \quad \Rightarrow \quad \frac{\partial \log c_{ZZ}}{\partial \tau} = -\frac{1}{\tau_Z}$$

The decorrelation timescale is just the damping scale

- **Sharp peak limit:** $\delta_\omega \ll \tau_Z^{-1}$

$$Z^*Z(\omega) \approx \frac{Ae^{-\omega^2/\delta_\omega^2}}{\tau_Z^{-2}} \quad \Rightarrow \quad C_{ZZ}(\tau) = A\tau_Z^2 e^{-\delta_\omega^2 \tau^2} \quad \Rightarrow \quad \frac{\partial \log c_{ZZ}}{\partial \tau} = -2\delta_\omega \tau \neq f(\tau_Z)$$

The decorrelation timescale is independent of friction. Since in the linear feedback model $\frac{\partial \log c_{ZZ}}{\partial \tau} = b - \frac{1}{\tau_Z} \neq f(\tau_Z)$, the eddy feedback b must scale as friction!



$$\tau_Z^{-1} = 0.12 \text{ days}^{-1} \quad \delta_\omega \sim 0.02 \text{ days}^{-1}$$

$\delta_\omega \ll \tau_Z^{-1}$ Relevant limit is **sharp-peak limit**.

$$Z^*Z = \frac{M^*M}{\omega^2 + \tau_Z^{-2}} \approx \tau_Z^2 M^*M$$

- M^*M peak much narrower than τ_Z^{-1}
- Z^*Z has same structure as M^*M

Physically, when the forcing varies on timescales much slower than friction, friction affects the amplitude of the response, but not its temporal structure.

(The important hard question is, what determines this characteristic scale δ_ω ?)

Conclusions

1. Dominant mode of extratropical variability:
annular modes \leftrightarrow jet shifts
2. Eddy feedback makes long-lag decay of jet shift anomalies slower than frictional timescale \Rightarrow enhanced persistence
3. Internal variability of eddy heat & momentum fluxes in the two-layer QG model is consistent with baroclinic feedback
4. Eddy feedback increases with friction in the two-layer model
5. This is *not* due to an enhanced baroclinic feedback but to the fact that the eddy forcing of the zonal index varies on timescales much longer than friction!

Thank you!!!