# Ribbon turbulence

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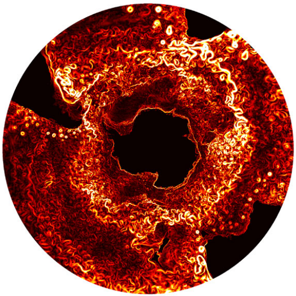
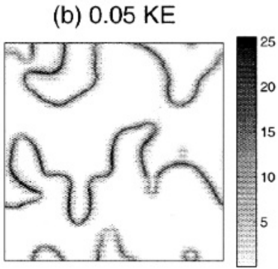
# Sharp and meandering jets



Surface oceanic currents.
*From ECCO2*




Large friction, baroclinic turbulence
*From Arbic & Flierl, JPO 04*

**Can we understand the spontaneous emergence and the dynamics of these ribbons ?**
**Statistical Mechanics approach**

# Sharp and meandering jets

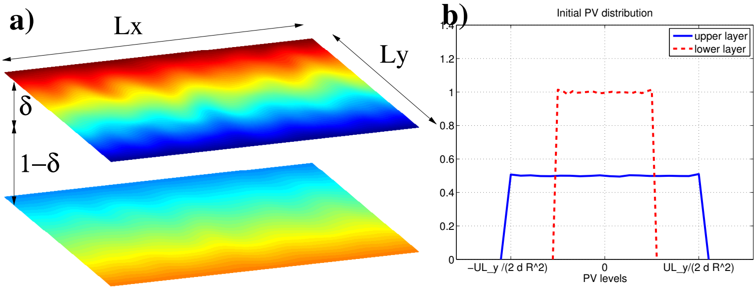
1. Previous interpretation of sharp jets as **equilibrium states of 1-1/2 layer QG model** (one active layer above a deeper layer at rest) *Bouchet Sommeria JFM 02, Venaille Bouchet JPO 11*

2. Yet **baroclinic instability requires at least two layers**. Previous work on the vertical structure of geostrophic turbulence:
   - stratification *Smith Vallis JPO 01 02*
   - **bottom friction** *Thompson Young JPO 06, Arbic Flierl JP0 04*
   - bottom topography, beta effect,...

# Outline

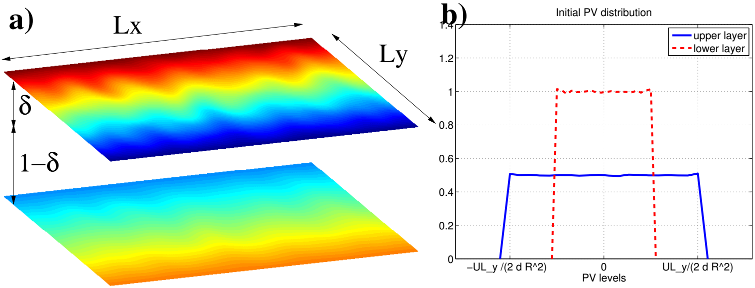
1. Numerical simulations of two-layer baroclinic turbulence
2. 1-1/5 QG turbulence in the large bottom friction limit
3. Statistical mechanics interpretation of the ribbons
4. The role of baroclinic instability

# 2 layers QG flow in a channel



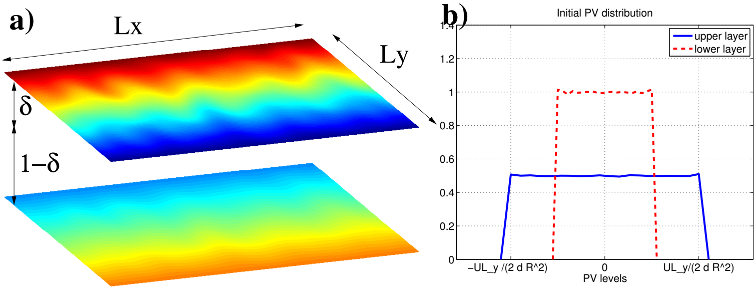

$$\partial_t q_1 + J(\Psi_1, q_1) = 0,$$
$$\partial_t q_2 + J(\Psi_2, q_2) = -r\nabla^2\Psi_2$$

# 2 layers QG flow in a channel



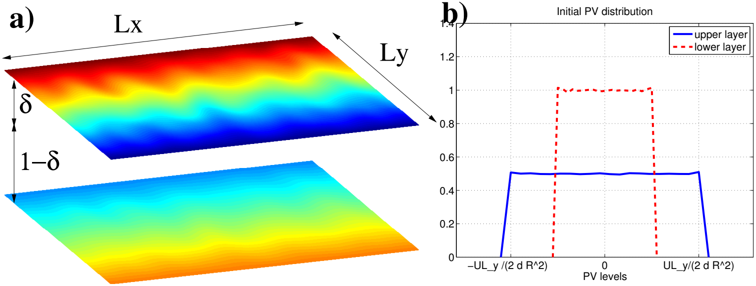

$$
\begin{aligned}
q_1 &= \nabla^2 \Psi_1 + \frac{\Psi_2 - \Psi_1}{\delta R^2}, \\
q_2 &= \nabla^2 \Psi_2 + \frac{\Psi_1 - \Psi_2}{(1-\delta) R^2}
\end{aligned}
$$

# 2 layers QG flow in a channel




$$\Psi_1 = -Uy + \psi_1 \ ,$$
$$\Psi_2 = 0 + \psi_2$$

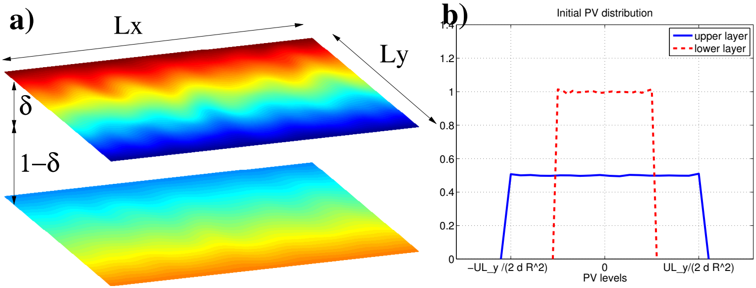
# 2 layers QG flow in a channel




$$q_1 = \nabla^2\psi_1 + \frac{\psi_2 - \psi_1}{\delta R^2} + \frac{U}{\delta R^2} y \,,$$

$$q_2 = \nabla^2\psi_2 + \frac{\psi_1 - \psi_2}{(1-\delta)\, R^2} - \frac{U}{(1-\delta)\, R^2} y \,.$$

# Dynamical invariants




$$\partial_t q_1 + J(\psi_1 - Uy, q_1) = 0,$$
$$\partial_t q_2 + J(\psi_2, q_2) = -r\nabla^2\psi_2$$

**Conservation of the global distribution of potential vorticity levels in the upper layer**

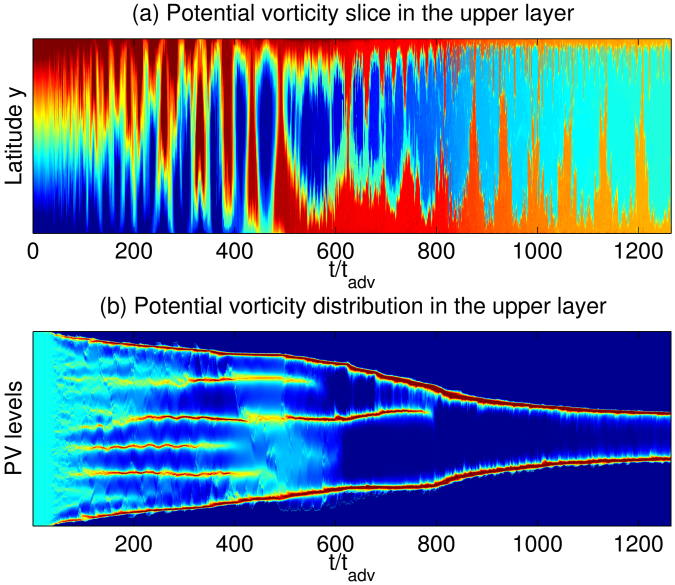
This is true for a channel geometry, not for a doubly periodic domain

# Parameters

1. Bottom friction $rU/R$
2. Rossby radius $R/L_y$
3. Vertical aspect ratio $\delta$
4. Horizontal aspect ratio $L_x/L_y$

**What is the effect of bottom friction ?**

# Emergence of the ribbons



# Energy budget for the perturbation

$$E = \frac{1}{2}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y\, \delta\left(\nabla\psi_1\right)^2 + (1-\delta)\left(\nabla\psi_2\right)^2 + \frac{\left(\psi_1-\psi_2\right)^2}{R^2}$$

$$\boxed{\frac{R}{U}\frac{\mathrm{d}}{\mathrm{d}t}E = \frac{1}{R}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y\ \psi_1\partial_x\psi_2 - (1-\delta)\frac{rR}{U}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y\ \left(\nabla\psi_2\right)^2}$$

This holds for both the doubly periodic and the channel geometry.

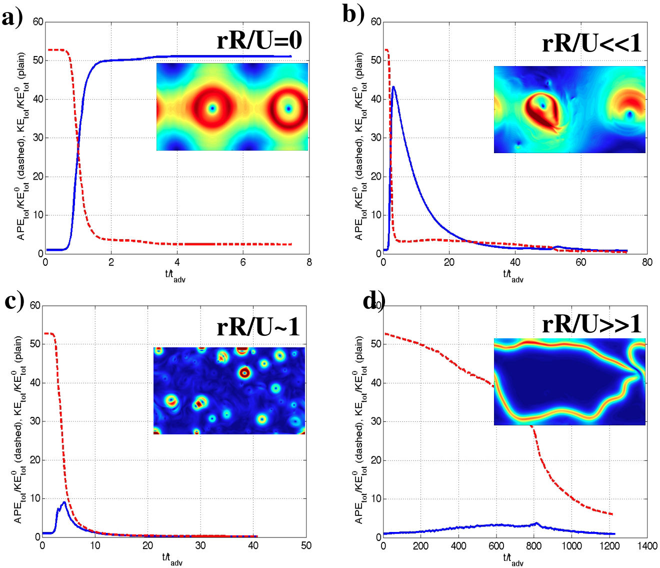
# Energy budget for perturbation plus mean flow

$$E_{tot} = \frac{1}{2}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y\, \delta\left(\nabla\left(\psi_1 - Uy\right)\right)^2 + (1-\delta)\left(\nabla\psi_2\right)^2 + \frac{\left(\psi_1 - Uy - \psi_2\right)^2}{R^2}$$

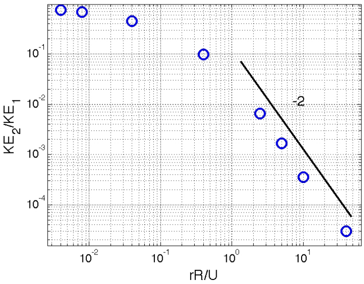
$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}E_{tot} = -(1-\delta)r\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y\ \left(\nabla\psi_2\right)^2\ .}$$

This holds only for the channel geometry.

# Temporal evolution of the total energy



# Large bottom frictions




$$\frac{\psi_2}{\psi_1} \sim \left(\frac{rR}{U}\right)^{-2}$$

$$\frac{R}{U}\frac{\mathrm{d}}{\mathrm{d}t}E = \frac{1}{R}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y \;\; \psi_1 \partial_x \psi_2 - (1-\delta)\frac{rR}{U}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y \;\; (\nabla\psi_2)^2$$

# Dynamics

$$\Psi_1 = \psi_1 - Uy, \quad \psi_2 \ll \psi_1$$

$$\partial_t q_1 + J\left(\Psi_1, q_1\right) \quad = \quad 0,$$

$$q_1 \quad = \quad \nabla^2 \Psi_1 - \frac{\Psi_1}{\delta R^2},$$

**For a given energy and potential vorticity distribution, can we predict the large scale flow ?**

# Equilibrium states

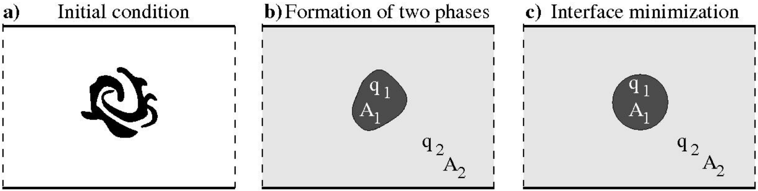
- $\partial_t q_1 + J(\Psi_1, q_1) = 0$
- **"Incompressibility constraint:"** conservation of the area $A_\sigma$ associated with each level $\sigma$ of PV
- **Energy conservation:** $E = \frac{1}{2}\int \mathrm{d}\mathbf{r}\ \left(\nabla\Psi_1\right)^2 + \frac{\Psi_1^2}{\delta R^2}$
- Equilibrium theory by *Miller (1990), Robert Sommeria (1991)*

**The observed flow is the most probable one among all the states that satisfy the constraints of the dynamics**

# Small R limit

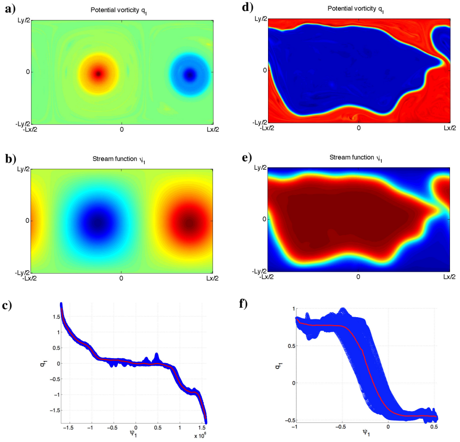
*Bouchet Sommeria JFM 02* studied one class of equilibria the limit $\delta^{1/2}R \ll L_y$

$$q_1 = \Delta\Psi_1 - \frac{\Psi_1}{\delta R^2}$$

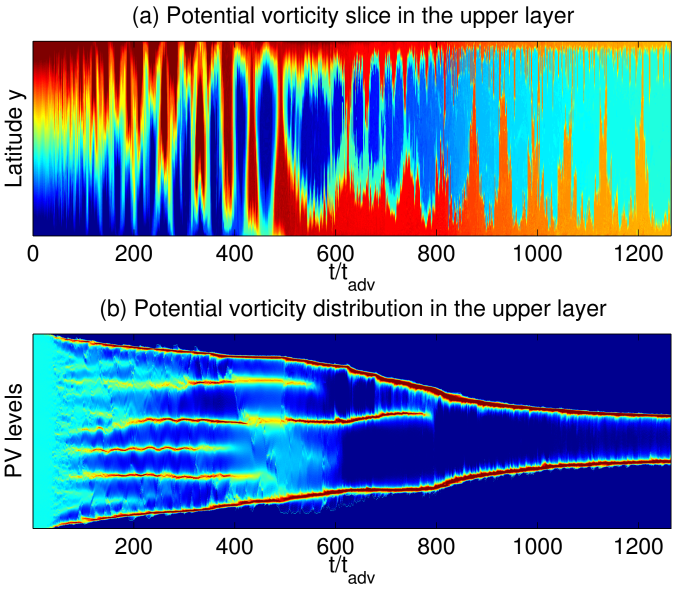



- **Formation of different phases of homogenized PV with strong jets at their interface.**
- **Interfaces "cost" free energy**
- *tanh*-relation between $q_1$ and $\Psi_1$

# q-psi relation

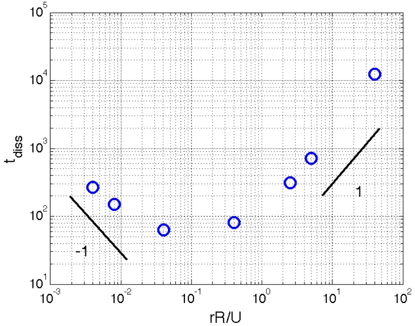


# Emergence of the ribbons



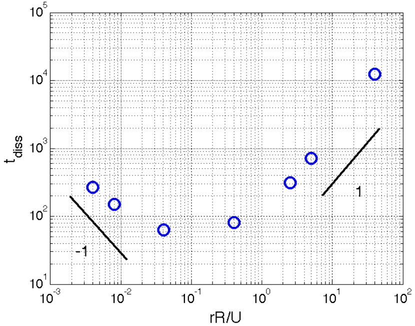
# Time scale for energy dissipation

**Increasing bottom friction decreases the time scale for energy dissipation when $rU/R \gg 1$ !**




Dimensional analysis: $t_{diss} \sim r^{-1} f\left(\frac{rR}{U}, \frac{R}{L_y}, \delta, \frac{L_x}{L_y}\right)$

# Time scale for energy dissipation

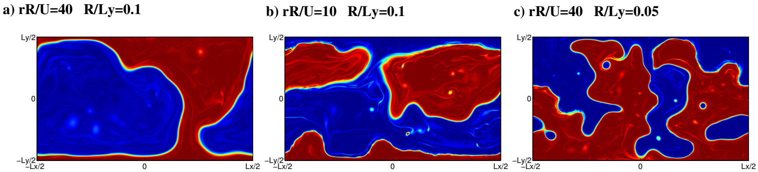



$$t_{diss} \sim \frac{1}{r}\left(\frac{rR}{U}\right)^2 \frac{L_y}{R}$$

$$\frac{R}{U}\frac{\mathrm{d}}{\mathrm{d}t}E = \frac{1}{R}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y\ \psi_1 \partial_x \psi_2 - (1-\delta)\frac{rR}{U}\int_{\mathcal{D}} \mathrm{d}x\mathrm{d}y\ \left(\nabla\psi_2\right)^2$$

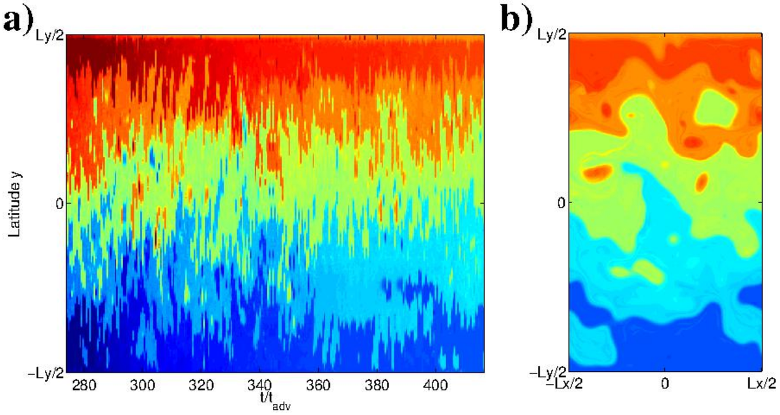
# The interface length

- Blobs of homogenized PV of size $L_{blob}$ with $\Psi_1 \sim U L_y$.
- Inertial time scale $t_{relax}$ estimated using planetary QG $\partial_t(\Psi_1/\delta R^2) = J(\Psi_1, \nabla^2 \Psi_1)$
- Baroclinic instability destabilizes the interface $t_{diss}$




$$L_{blob} \sim L_y \left(\frac{rR}{U}\right)^{1/4} \left(\frac{R}{L_y}\right)^{1/2}$$

# PV staircases



# Conclusion

1. Ribbons are sharp jets between regions of homogenized potential vorticity
2. The appear in a large friction limit of baroclinic turbulence.
3. Their dynamics results from a competition between a tendency to reach an equilibrium state and baroclinic instability of the jets.
4. Energy dissipation time scale increases with bottom friction coefficient
5. Formation of PV staircases in the early evolution of the flow

# Cascade phenomenology

**1.5 layer QG**
$\partial_t q_1 + J(\Psi_1, q_1) = 0$ with $q_1 = \zeta_1 - \frac{\Psi_1}{\delta R^2}$, $\zeta_1 = \Delta\Psi_1$

**Small scales** $\ll \delta^{1/2} R$
$\partial_t \zeta_1 + J(\Psi_1, \zeta_1) = 0$

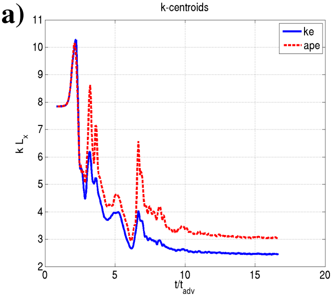
$KE_1 = -\int \mathrm{d}\mathbf{r}\ \zeta_1 \Psi_1$ cascades towards **large** scales.
$Z_1 = \int \mathrm{d}\mathbf{r}\ \zeta_1^2$ cascades towards **small** scales.

**Large scales** $\gg \delta^{1/2} R$
$\partial_\tau \Psi_1 + J(\zeta_1, \Psi_1) = 0 \quad \tau = R^2 t$

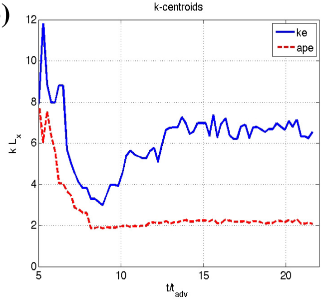
$KE_1 = -\int \mathrm{d}\mathbf{r}\ \zeta_1 \Psi_1$ cascades towards **small** scales.
$APE = \int \mathrm{d}\mathbf{r}\ \Psi_1^2$ cascades toward **large** scales.

# Centroids

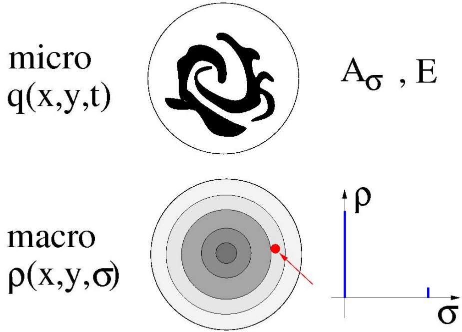
a)




b)



# Stat. Mech.



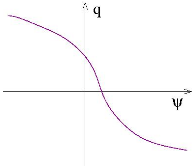

- To find $\overline{q} = \int \mathrm{d}\sigma\ \sigma\rho$
- with $\overline{q} - \beta y = \Delta\psi - \psi/R^2$

- Maximize $\boldsymbol{\mathcal{S} = -\int \mathrm{d}x\mathrm{d}y\mathrm{d}\sigma\ \rho \ln \rho}$
- With constraints expressed in term of $\rho$.

# Critical points are dynamical equilibria

$$\delta \mathcal{S} - \beta_T \delta \mathcal{E} + \int \mathrm{d}\sigma \; \alpha \delta \mathcal{A}_\sigma = 0 \quad \text{gives a relation} \quad \boldsymbol{\rho(\psi)}$$

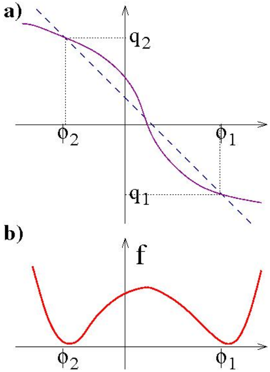
$$\overline{q}(x,y) = \int \mathrm{d}\sigma \; \sigma \rho = g\left(\overline{\psi}\right)$$




**Critical points could be entropy maxima, minima, or saddle**

# $R$ Small

- $q - \beta y = R^2 \Delta\phi - \phi \; (\phi = \psi / R^2)$
- Minimize the free energy
  $\mathcal{F} = \int_D d\mathbf{r} \left[ \frac{R^2 (\nabla\phi)^2}{2} + f(\phi) - \phi\beta y \right]$
- With constraints $\mathcal{A} = \int_D \mathrm{dxdy} \; \phi$
- Critical points satisfy
  $f'(\overline{\phi}) = \overline{q}(\phi) + 2\phi - \alpha$



# Thermodynamical analogy

Van der Waals Cahn Hilliard variational problem describing 1st order phase transitions.

$$\min \left\{ \int_D \mathrm{d}x\mathrm{d}y \, \left[ \frac{R^2 (\nabla \phi)^2}{2} + f(\phi) \right] \, \Big| \, \int_D \mathrm{d}x\mathrm{d}y \, \phi = \mathcal{A} \right\}$$