Mesoscale Eddy Energy Locality in an Idealized Ocean Model

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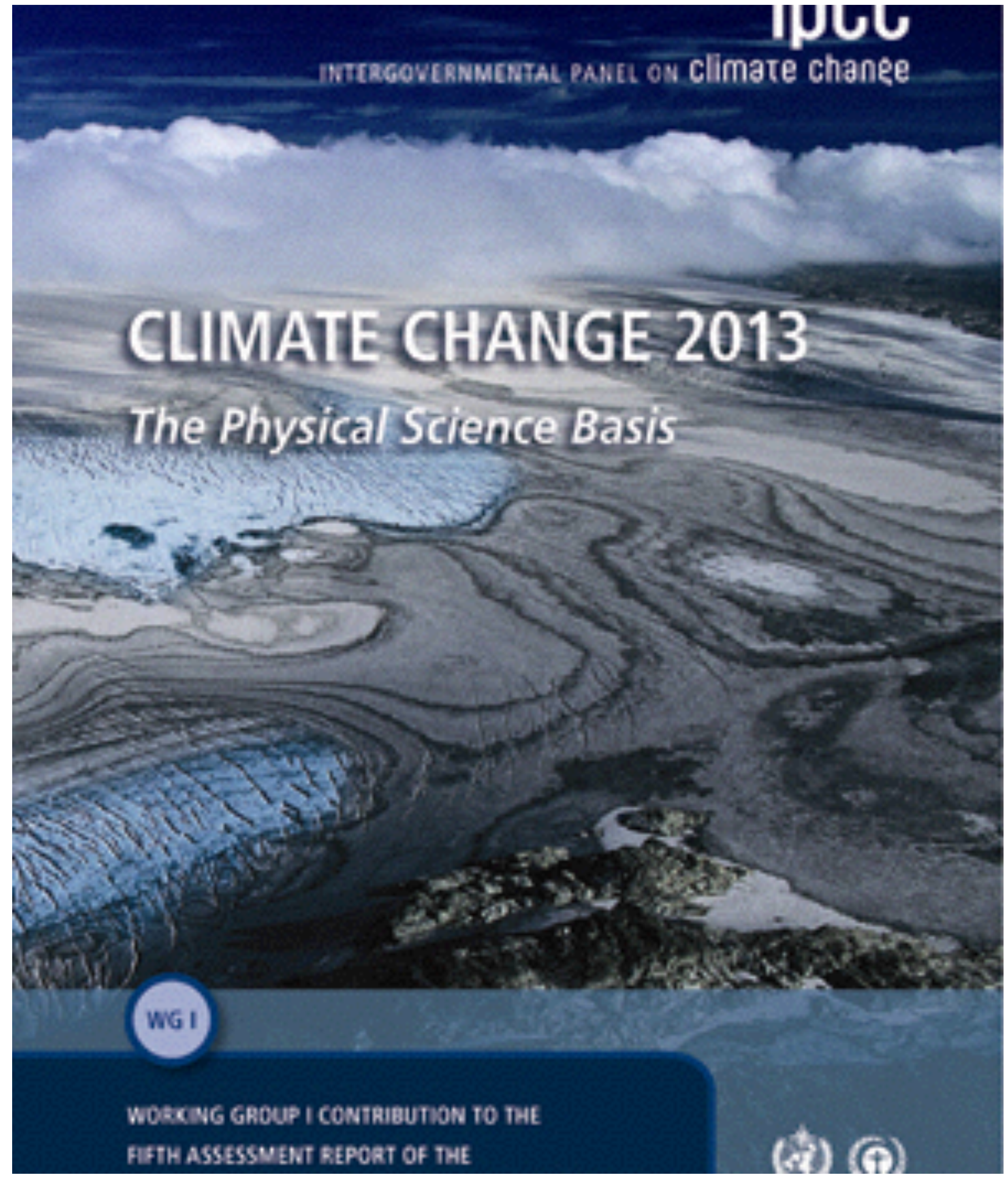
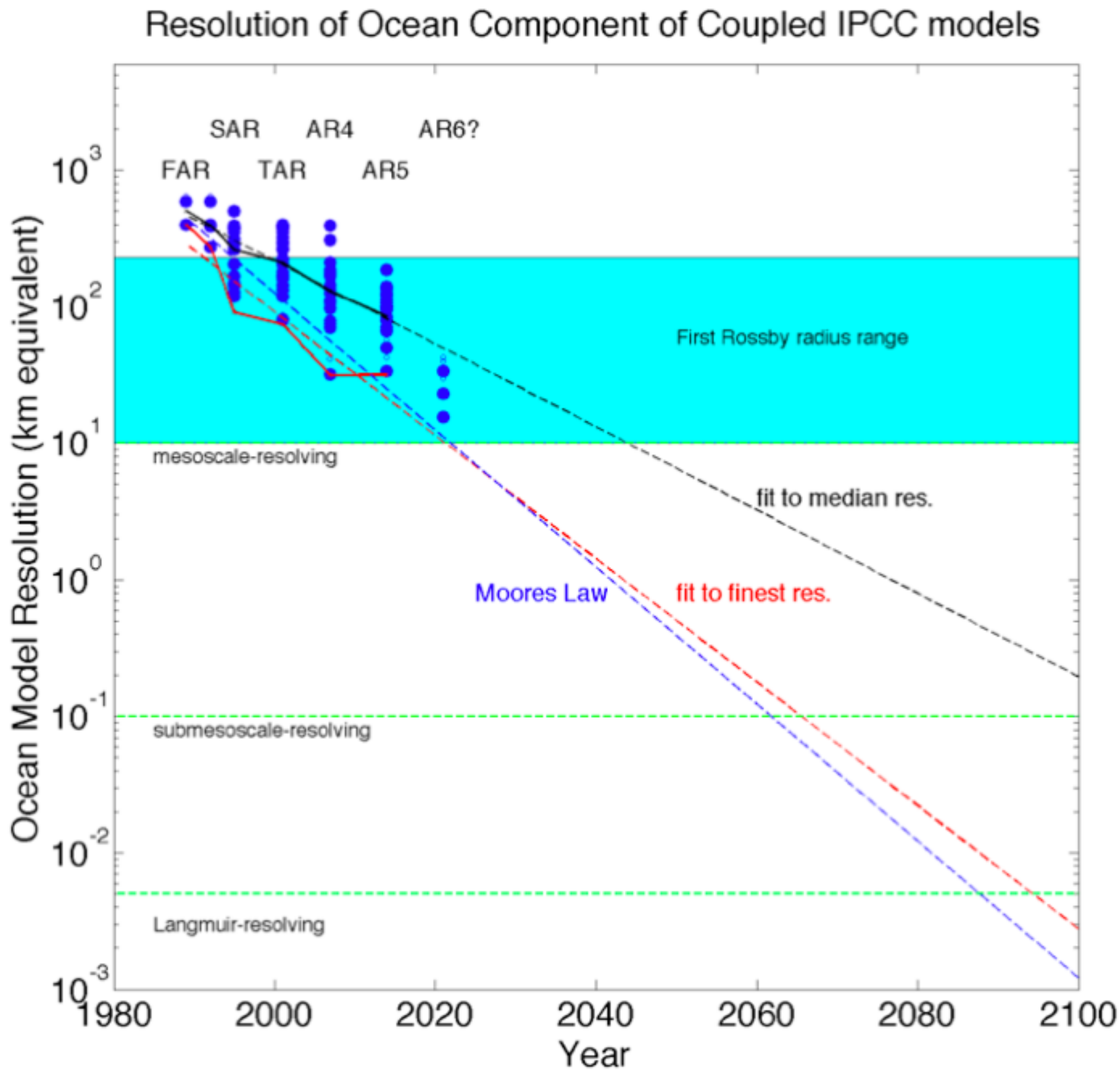
Workshop on Fundamentals of Climate, Atmosphere and Ocean Dynamics
MPI, Hamburg
12-14 May 2014

Introduction

- Simulations are an indispensable tool in climate science
- It is impossible to resolve all dynamically active scales in climate simulations
- **Parameterizations are the foundation**
- This talk relates to underlying assumptions in parameterizations of ocean eddies





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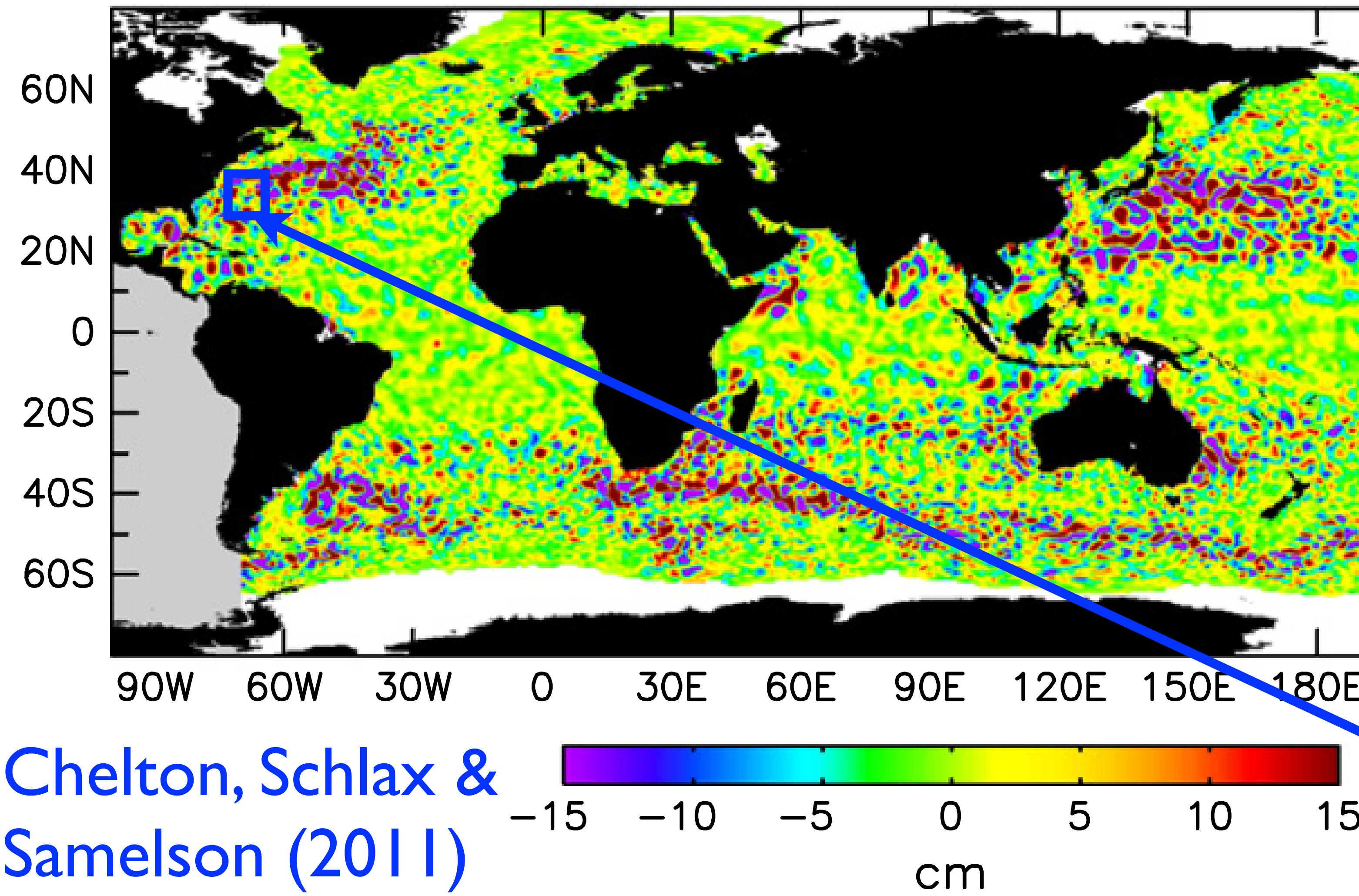
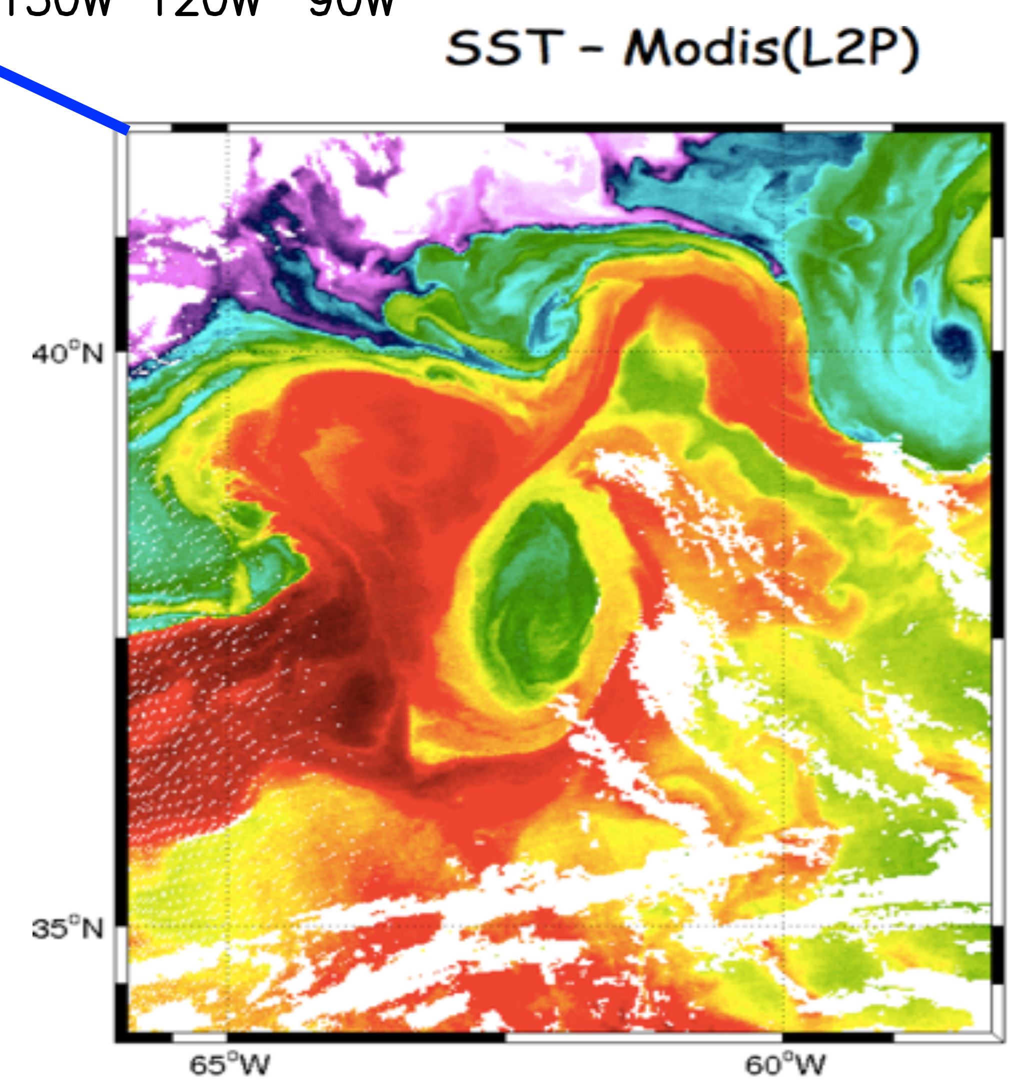


AVISO

Chelton, Schlax & Samelson (2011)



‘Eddies’ (vortices, filaments, fronts, ..) are everywhere, ranging in scales from ~100km down to 100m

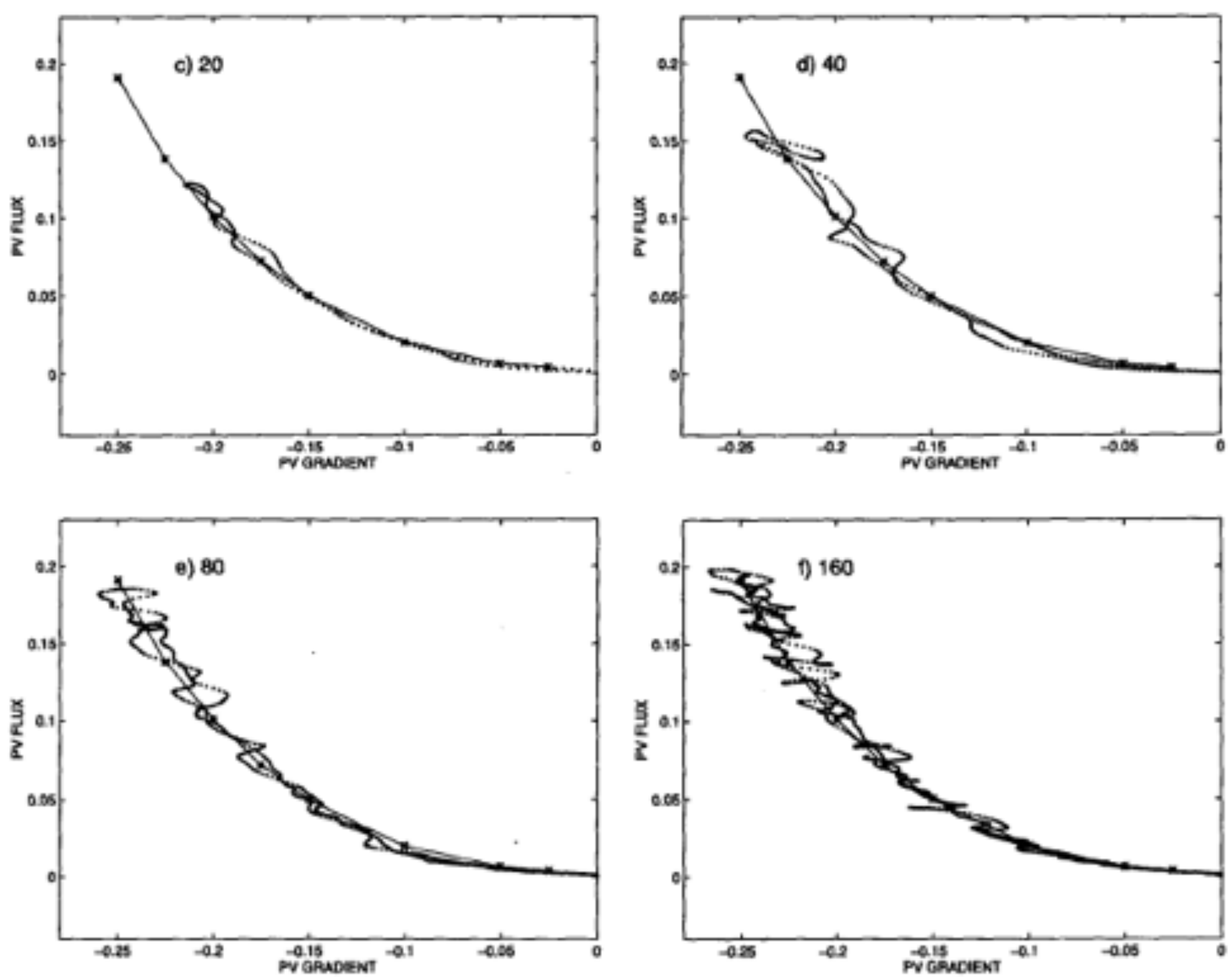
Eddy parameterizations are **theories** for four-dimensional eddy fluxes, in the absence of resolved eddies

Most parameterizations assume eddy fluxes are functions of **local** mean flow properties, e.g.

- Gent & McWilliams 90: Eddy buoyancy flux ~ isopycnal gradient, with diffusivity κ
- Held&Larichev 96, Visbeck etal 97, Cessi etal 08: Set $\kappa = v_e \times l_e$ with theories for eddy velocity and lengths dependent on local mean flow

Periodic QG models often used as parameterization theories, and sometimes this works:

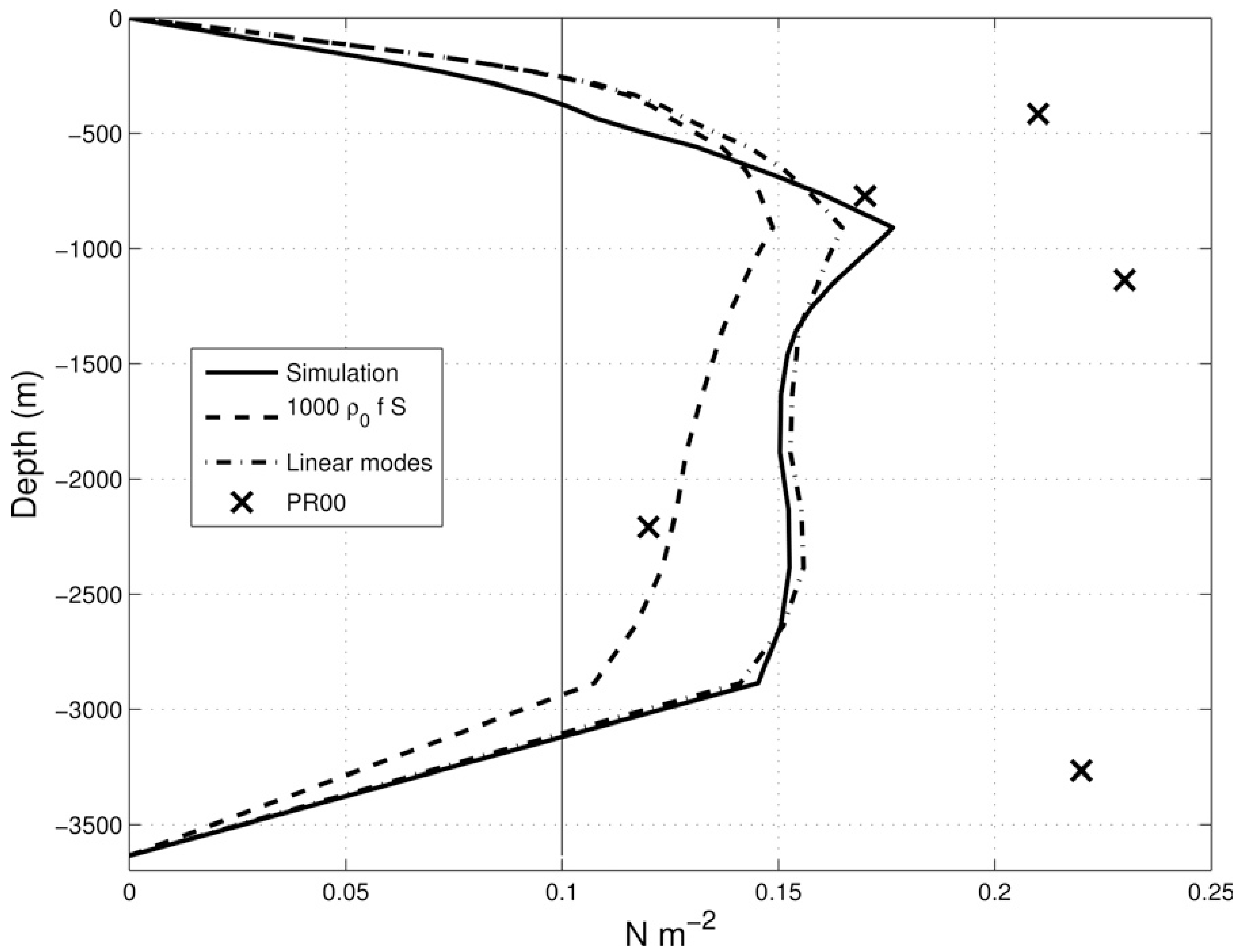
- Pavan&Held 96: compared the PV flux diagnosed from channel simulations and doubly-periodic simulations.



The local PV flux diagnosed from the channel simulations, shown as a function of the local mean PV gradient (dots) matches the relationship obtained from doubly-periodic simulations (solid).

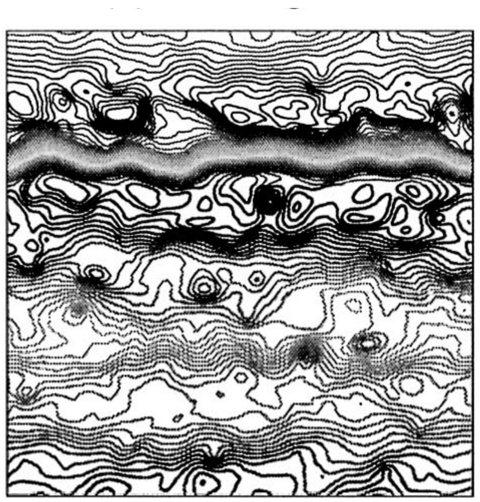
Periodic QG models often used as parameterization theories, and sometimes this works:

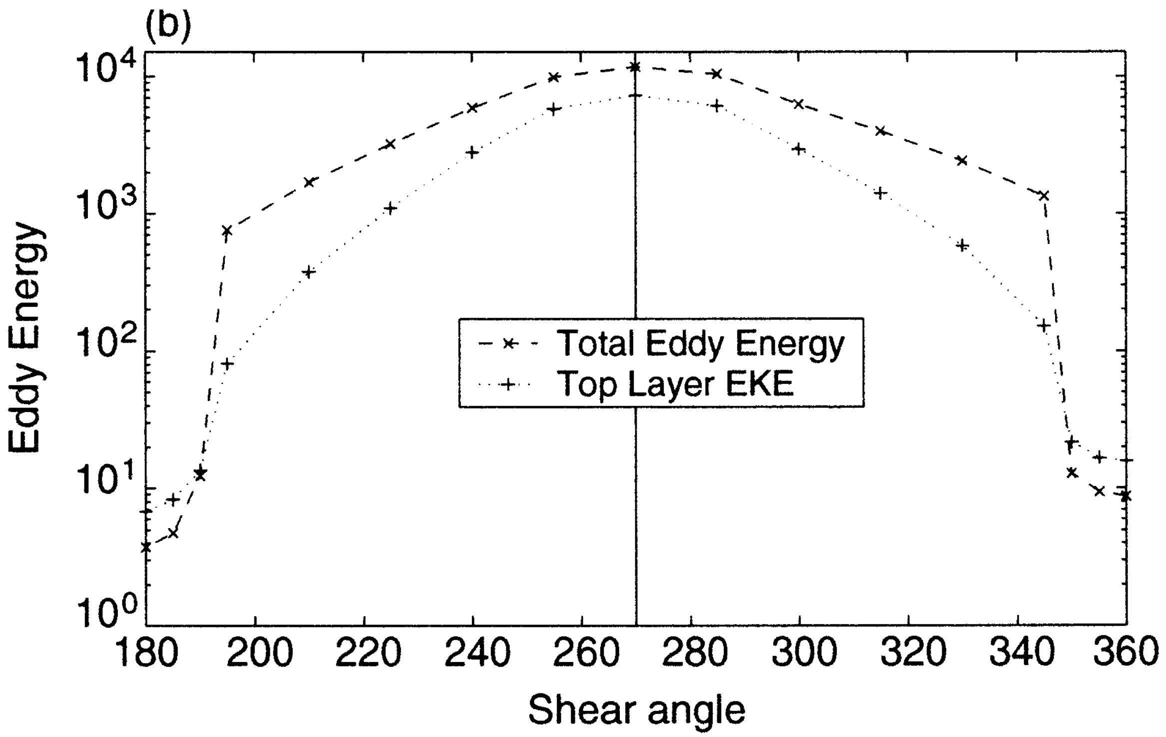
- Smith & Marshall 09: found agreement between the eddy fluxes diagnosed from doubly-periodic simulations using mean state from region in SO to those measured fluxes from moored array

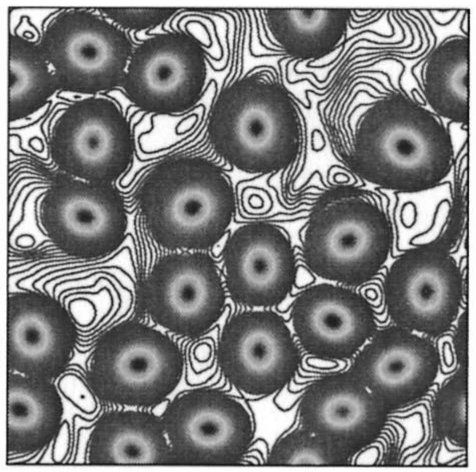


But sometimes periodic QG models can achieve **unphysical** states or fail to equilibrate

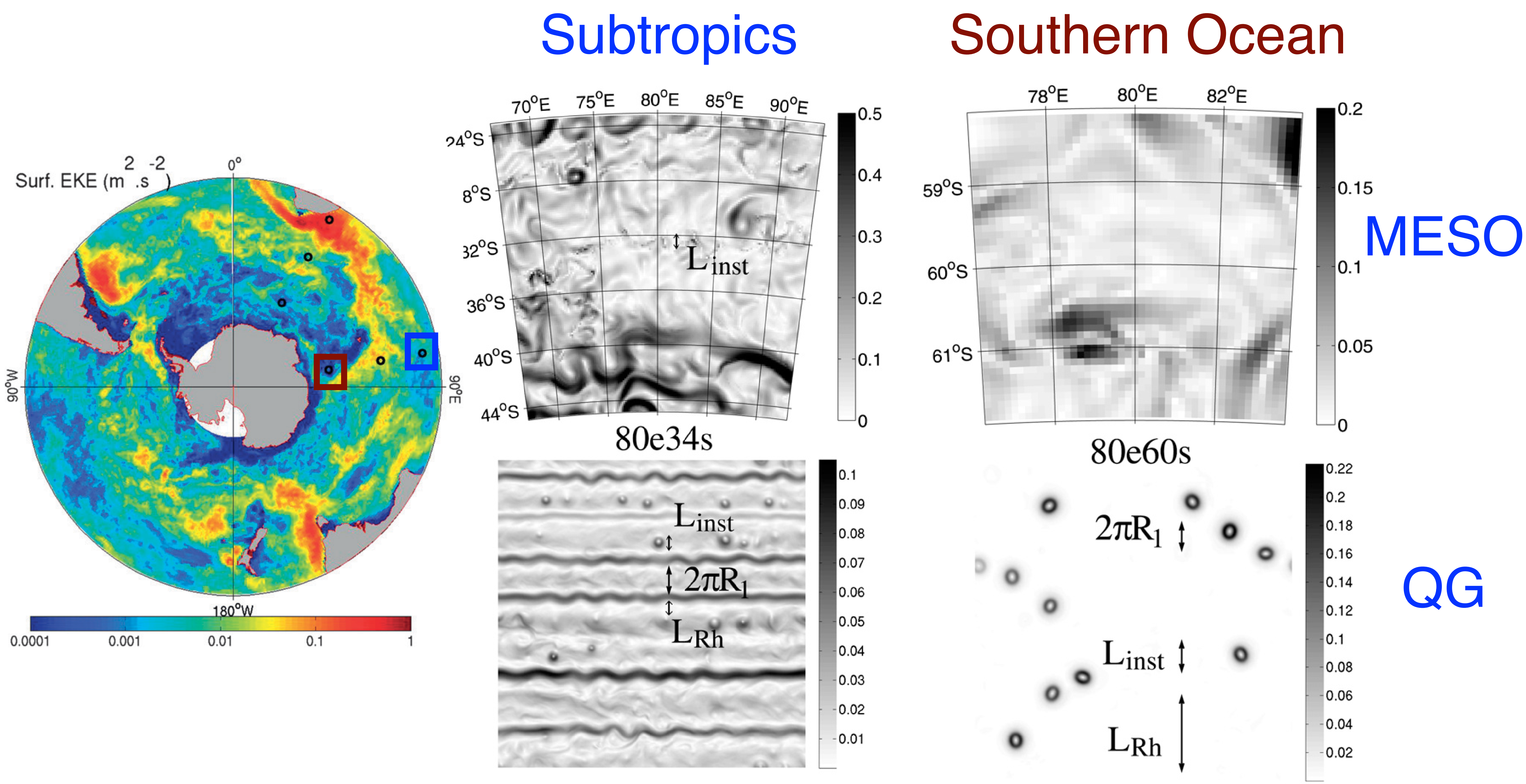
- Thompson & Young 06: f-plane simulations require excessive drag to equilibrate
- Spall 00, Arbic & Flierl 04, Smith 07: Non-zonal mean flow leads to shearing of PV by β jets, resulting in excessive eddy energy & weird states





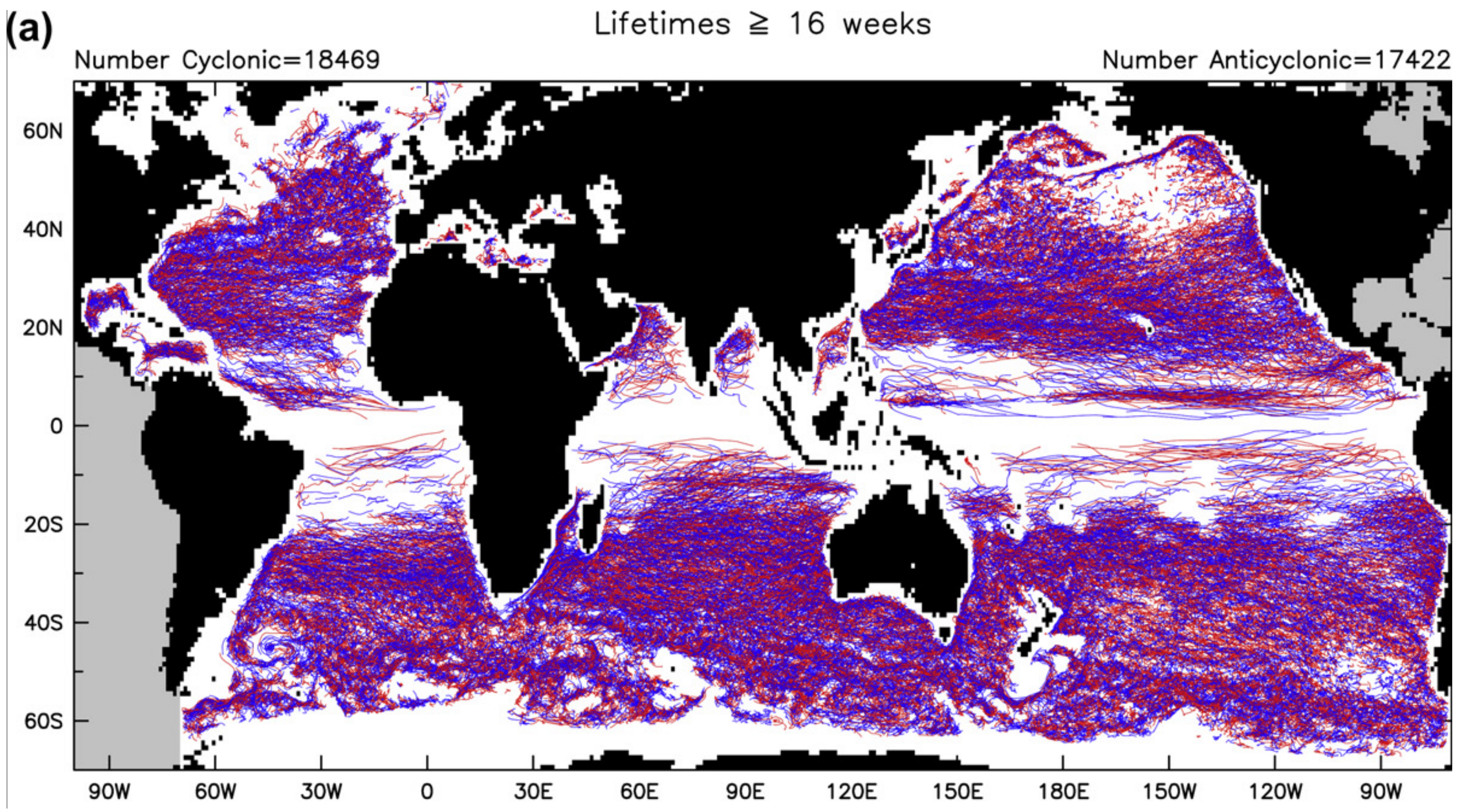


Venaille, Vallis & Smith 11: Compared eddy statistics in regions of eddy-permitting 1/6deg GFDL simulation of Southern Ocean to local QG simulations: some regions match up, others don't.



Venaille, Vallis & Smith 11:

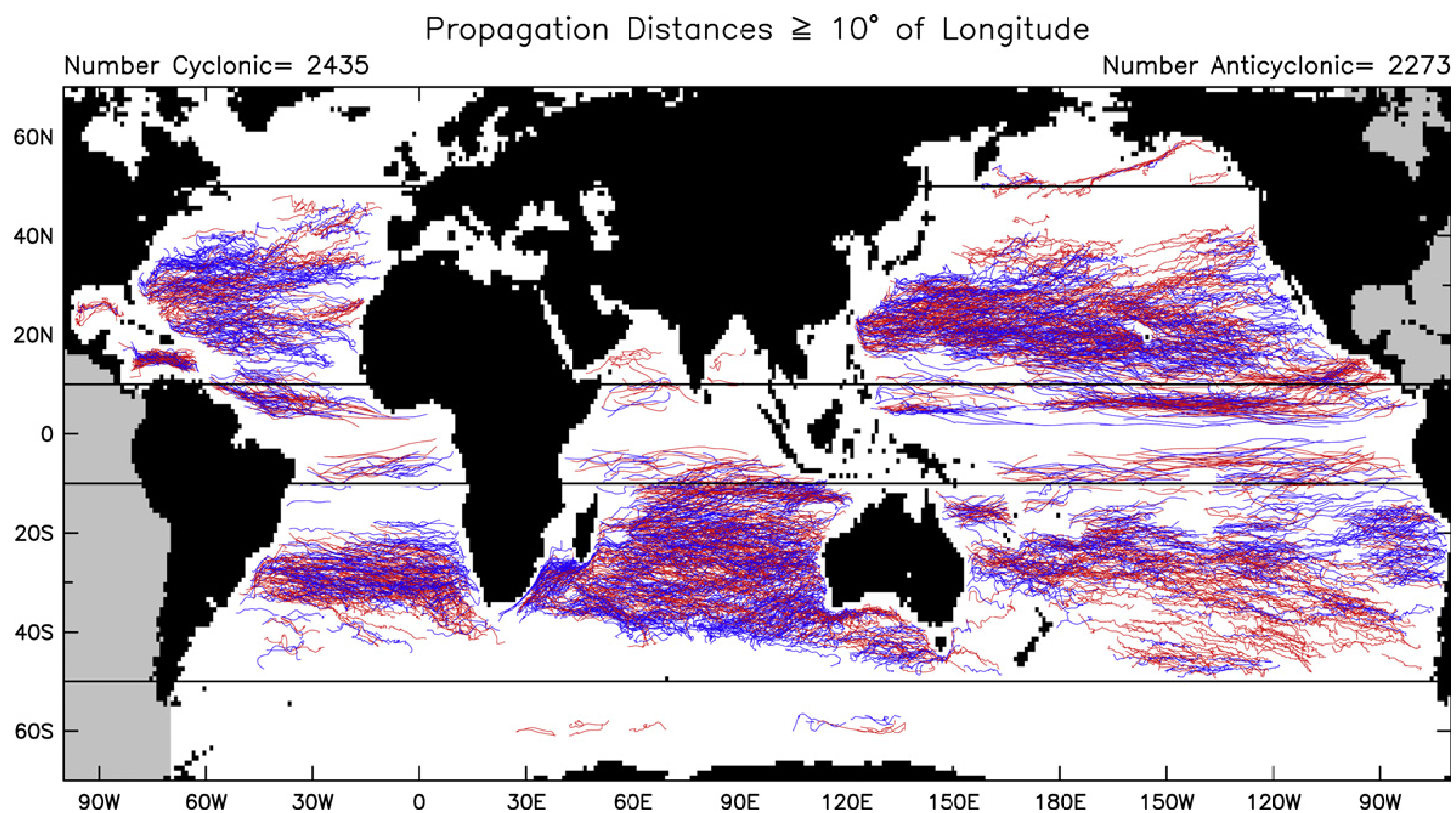
We conjecture that, in regions characterized by strong and fast instabilities (time scales from days to weeks), the length scale and the regime of self-organization can be interpreted with local, nonlinear quasigeostrophic simulations. However, because the simulations are local, the propagation of eddies away from the source is neglected, and, because the mean flow is imposed, possibly important interactions between eddies and mean flows are neglected. In regions characterized by weak, slow instabilities, the dynamics seem to be governed by eddies coming from more unstable regions. In some regions, artificially high bottom drag or thermal damping were necessary to equilibrate quasigeostrophic simulations, suggesting that the primitive equations are equilibrating by nonlocal mechanisms (e.g., the advection of the eddy field away from a region of instability) or possibly through ageostrophic sources of dissipation.



Chelton, Schlax & Samelson (2011)

AVISO SSH: coherent vortices (one type of eddy) are long-lasting - some over 1 year…

.. and in that time travel westward, many for over 1000km. Eddy energy not a function of local mean



Locality defined

Eddy energy can be generated (*G*), dissipated (*D*), and fluxed (*F*) from one place to another:

$$\partial_t E = G + F + D$$

The eddy energy budget of a region is **local** if generation balances dissipation in that region.

Almost all mesoscale eddy parameterizations are local

Eddy energy **non**-locality would suggest that nonlocal parameterizations might work better than local ones.

Prognostic eddy energy…

Parameterizations with prognostic eddy energy equations are common in engineering turbulence (e.g. K – ε models), but not in ocean modeling.

- Eden & Greatbatch 08, Marshall & Adcroft 10: Propose parameterizations with prognostic eddy energy equations.
- Grooms, Smith & Majda 12: Derive eddy energy equation from multiscale model for mesoscale-gyrescale interaction

Contents lists available at SciVerse ScienceDirect

Dynamics of Atmospheres and Oceans

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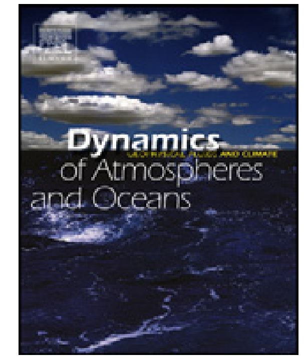


Dynamics of Atmospheres and Oceans 58 (2012) 95–107

Multiscale models for synoptic–mesoscale interactions in the ocean

Ian Grooms*, K. Shafer Smith, Andrew J. Majda

Primitive equations with scaling $\frac{L_{x,y}}{L_{X,Y}} \equiv \epsilon \ll 1 \quad L_{x,y} = L_d = NH/f_0.$

with $L_{X,Y}$ << planetary scale, and weak gyrescale buoyancy gradients $\overline{b}_0 = \overline{b}_0(z, \epsilon\delta X, \epsilon\delta Y)$ eventually leads to

$$\partial_T \langle E\rangle + \overline{\mathbf{u}}_0 \cdot \overline{\nabla}_h \langle E\rangle + \overline{\nabla}_h \cdot \langle \mathbf{u}_0' E\rangle - \overline{\nabla}_h \times \langle p_1'(\partial_t \mathbf{u}_0' + \overline{\mathbf{u}}_0 \cdot \nabla_h \mathbf{u}_0')\rangle = -\langle \mathbf{u}_0' \cdot (\mathbf{u}_0' \cdot \overline{\nabla}_h \overline{\mathbf{u}}_0)\rangle$$

$$- \left\langle \frac{\overline{\mathbf{u}_0' b_1'} \cdot (\overline{\nabla}_h \overline{b}_1 + \delta \overline{\overline{\nabla}}_h \overline{b}_0)}{\partial_z \overline{b}_0} \right\rangle + \overline{\mathbf{u}_0'|_{z=1} \cdot \boldsymbol{\tau}} - \overline{|\mathbf{u}_0'|^2|_{z=0}} + \overline{p_1|_{z=0}'(\overline{\mathbf{u}}_0 \cdot \nabla_h \eta_b')}.$$

JOURNAL OF PHYSICAL OCEANOGRAPHY

SEPTEMBER 2013

Mesoscale Eddy Energy Locality in an Idealized Ocean Model

IAN GROOMS, LOUIS-PHILIPPE NADEAU, AND K. SHAFER SMITH

Systematically measure **locality** in two-layer QG wind-driven gyre simulations:

- Simple setting keeps definitions and analysis focused on locality
- Allows comparison with periodic models using same dynamics (whereas in VVS, difference could be due to non-QG dynamics in PE model)

[Alternatives…]

- An alternative way to study the fluxes generated by mesoscale eddies is to diagnose them directly from an eddy-resolving data set. Recent examples include: Abernathey et al.13, Bachman & Fox-Kemper 13, Mana & Zanna 14 (Apologies to those neglected!)

- Once the flux terms are diagnosed, one looks for a relationship between the fluxes and the local properties of the mean flow. Bachman & Fox-Kemper also diagnosed the eddy KE directly, and found that using it improved the diagnostic performance of a parameterization.

Simulations

$$\partial_t q_1 + \nabla \cdot (\mathbf{u}_1 q_1) + \beta v_1 = F_w + A_h \nabla^4 \psi_1 \,,$$

$$\partial_t q_2 + \nabla \cdot (\mathbf{u}_2 q_2) + \beta v_2 = -r\nabla^2 \psi_2 + A_h \nabla^4 \psi_2 \,,$$

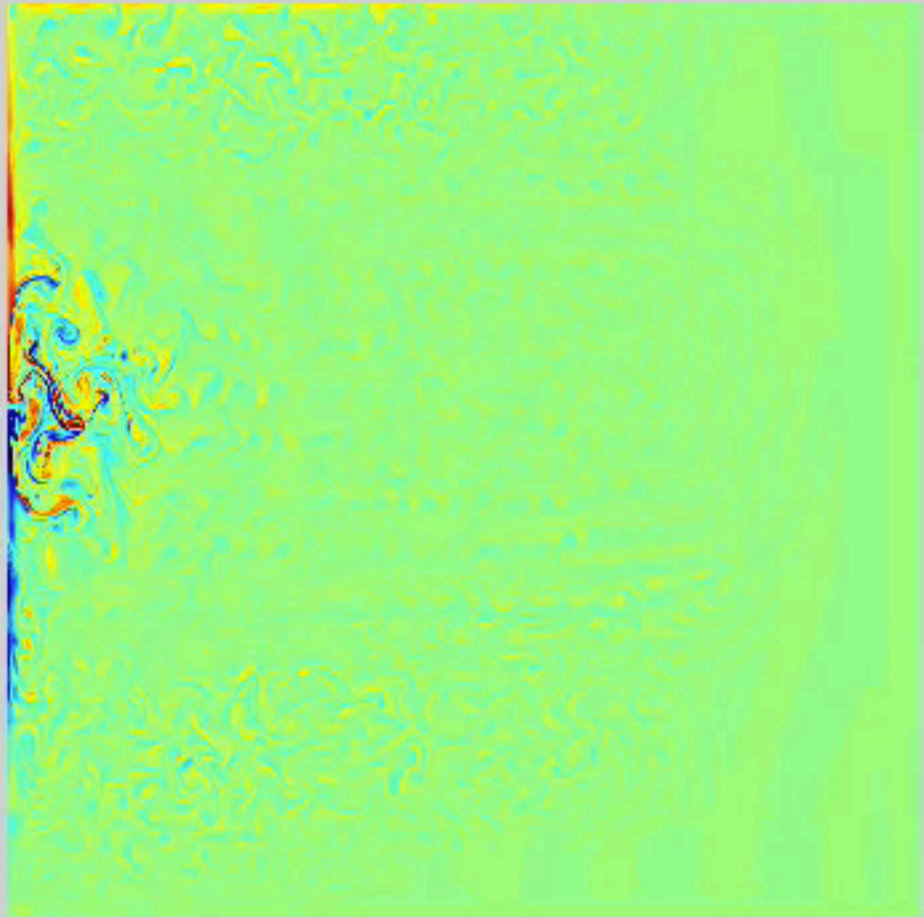
$$q_1 = \nabla^2 \psi_1 + F_1(\psi_2 - \psi_1) - F_0 \psi_1, \quad \text{and}$$

$$q_2 = \nabla^2 \psi_2 + F_2(\psi_1 - \psi_2)\,,$$

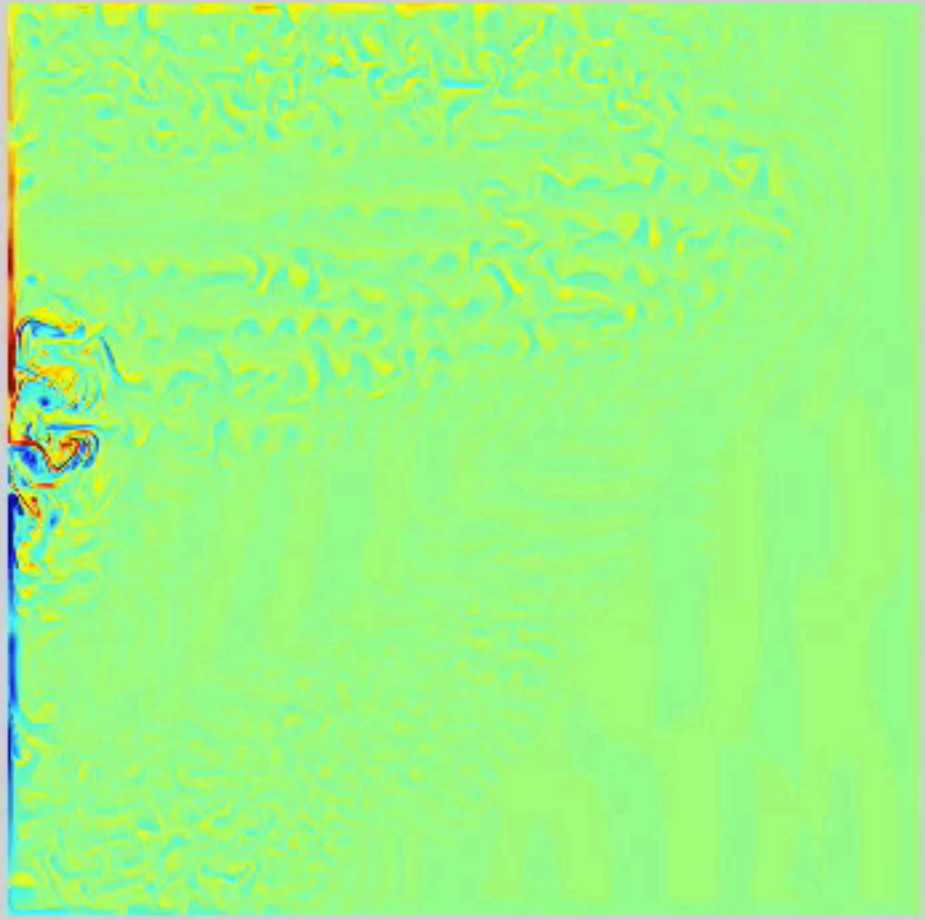
- Square domain, 6,144 km wide
- Deformation radius 20 km
- 8 km grid
- Layer depths 1 & 4 km
- Two different wind stress profiles: zonal and non-zonal

[Movies]

Zonal



Non-zonal



The filter defines the eddy

We need a way to separate the data set from our simulations into ‘mean’ and ‘eddy’ components.

Time and/or zonal averages are often used to define the ‘mean’ component. However, both methods fail to separate by spatial scale, so that the ‘mean’ has a small-scale part and the ‘eddies’ have a large-scale part.

Time mean also implies no energy transfer due to topography, but topography does transfer energy between scales.

You can use a Fourier series truncation to define large- and small-scale parts, even in an enclosed domain, but this is *nonlocal* because the basis functions have global support. It’s useful for global statistics (e.g. Nadiga & Straub 2010), but not for regional investigations.

We choose to define the large-scale mean using a spatial filter defined by the following elliptic inversion

$$(1 - L_f^2 \nabla^2)\overline{q}_i = q_i$$

[consider Fourier case…]

with boundary condition $\overline{q}_i = 0$. The small-scale eddy part is $q_i' = \overline{q}_i - q_i$. We'll use L_f = 180 km in analysis of simulations.

As with any spatial filter in an enclosed domain, it does not commute with differentiation

$$\nabla \overline{q} \neq \overline{\nabla q}.$$

For convenience we define the 'eddy' streamfunction $\tilde{\psi}_i$ by performing the PV inversion using q_i', and define eddy velocities using

$$\tilde{u}_i = -\partial_y \tilde{\psi}_i, \ \tilde{v}_i = \partial_x \tilde{\psi}_i.$$

Eddy equations

We apply the (high-pass) spatial filter to the QGPV equations to arrive at

$$\partial_t q_1' = -\nabla \cdot (\mathbf{u}_1 q_1)' - \beta \tilde{v}_1 + F_w' + A_h \nabla^4 \tilde{\psi}_1 + \mathrm{Error}_1$$

$$\partial_t q_2' = -\nabla \cdot (\mathbf{u}_2 q_2)' - \beta \tilde{v}_2 - r\tilde{\omega}_2 + A_h \nabla^4 \tilde{\psi}_2 + \mathrm{Error}_2$$

The "Error_i" result from formally commuting the spatial filter with derivatives. We track these errors in the code, and they are confined to a thin layer near the boundaries.

Local eddy energy budget

We derive the eddy energy equation by multiplying the eddy QGPV equations by $\tilde{\psi}_i$ and performing some simplifications. The result is

$$\partial_t \tilde{E} = G + F + D + \chi$$

- $\tilde{E} =$ eddy energy (kinetic plus potential)
- $G =$ 'generation' through interaction with the large-scale mean (can be negative, indicating upscale transfer)
- $F =$ all nonlocal terms, defined as the divergence of a flux
- $D =$ net dissipation due to Ekman drag and viscosity
- $\chi =$ terms resulting from commuting the filter with derivatives

The generation term has the form

$$G = \rho_0 \sum_i \{H_i[\tilde{u}_i(v_i q_i)' - \tilde{v}_i(u_i q_i)']\} - \rho_0 H_1 \tilde{\psi}_1 F_w'$$

The dissipation term has the form

$$D = -\rho_0 r H_2 |\tilde{\mathbf{u}}_2|^2 - \rho_0 A_h \sum_i H_i \tilde{\omega}_i^2$$

The nonlocal flux-divergence term has the form

$$F = \rho_0 \nabla \cdot \left\{ \frac{rH_2}{2} \nabla(\tilde{\psi}_2)^2 + A_h \mathbf{F}_{A_h} + \sum_i H_i [\tilde{\psi}_i \partial_t \nabla \tilde{\psi}_i + \tilde{\psi}_i (\mathbf{u}_i q_i)' + \beta \hat{\mathbf{x}} \tilde{\psi}_i^2 / 2] \right\}.$$

Diagnosis of energy budget

The next slide shows the four terms in the eddy energy budget, averaged over 10 years.

The budget is well-defined instantaneously; a time average simply smoothes out the results.

There is a very large variation in the size of the terms in the budget across the domain. To improve clarity we show the terms in the budget scaled by the eddy energy, e.g.

$$G/\tilde{E}.$$

Errors χ are small everywhere except near boundaries

Scaled generation rate

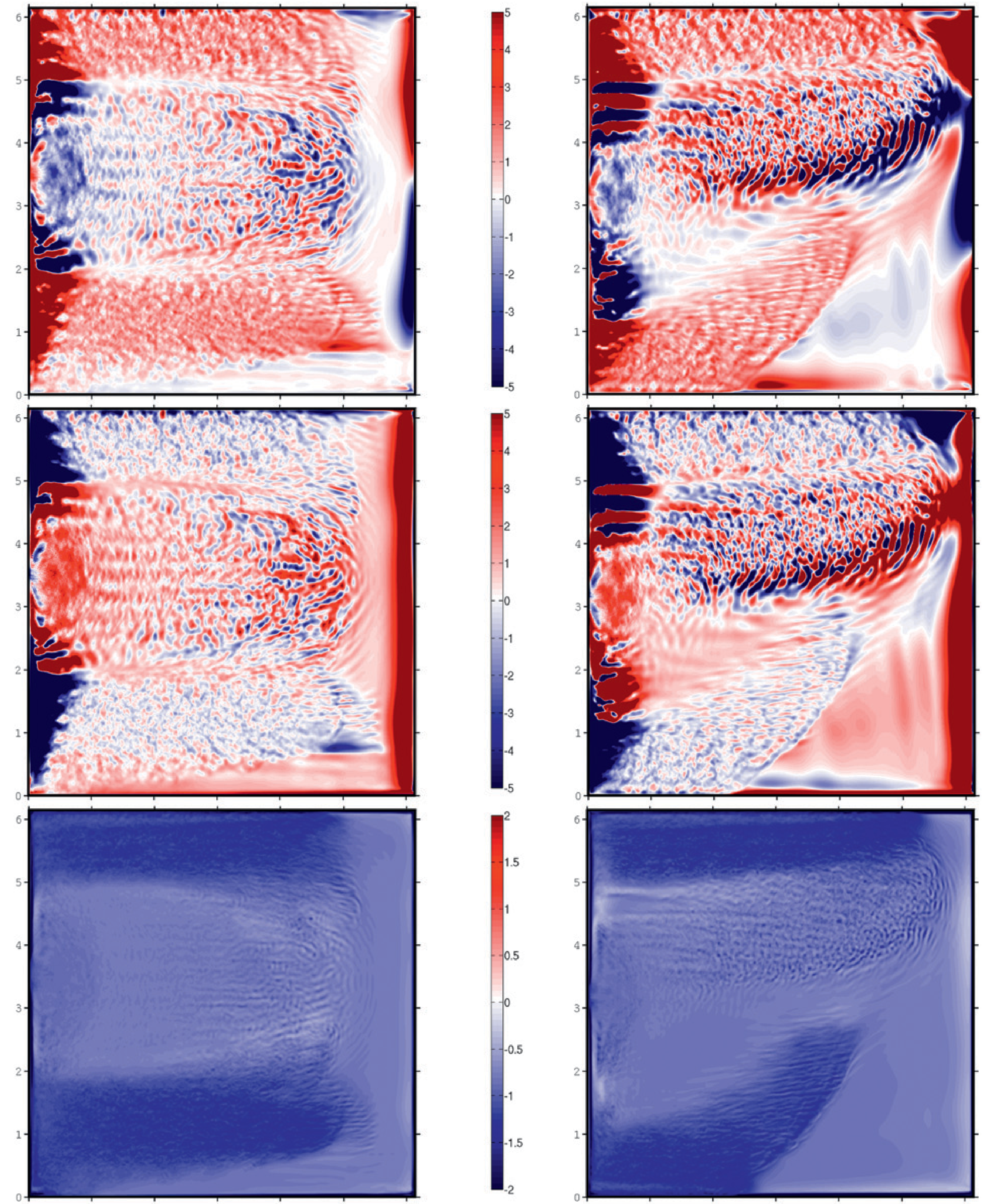
$\langle G \rangle_t / \langle \tilde{E} \rangle_t$

Scaled non-local flux rate

$\langle F \rangle_t / \langle \tilde{E} \rangle_t$

Scaled dissipation rate

$\langle D \rangle_t / \langle \tilde{E} \rangle_t$



Coarse-grained non-locality

[Average budget over 384km boxes]

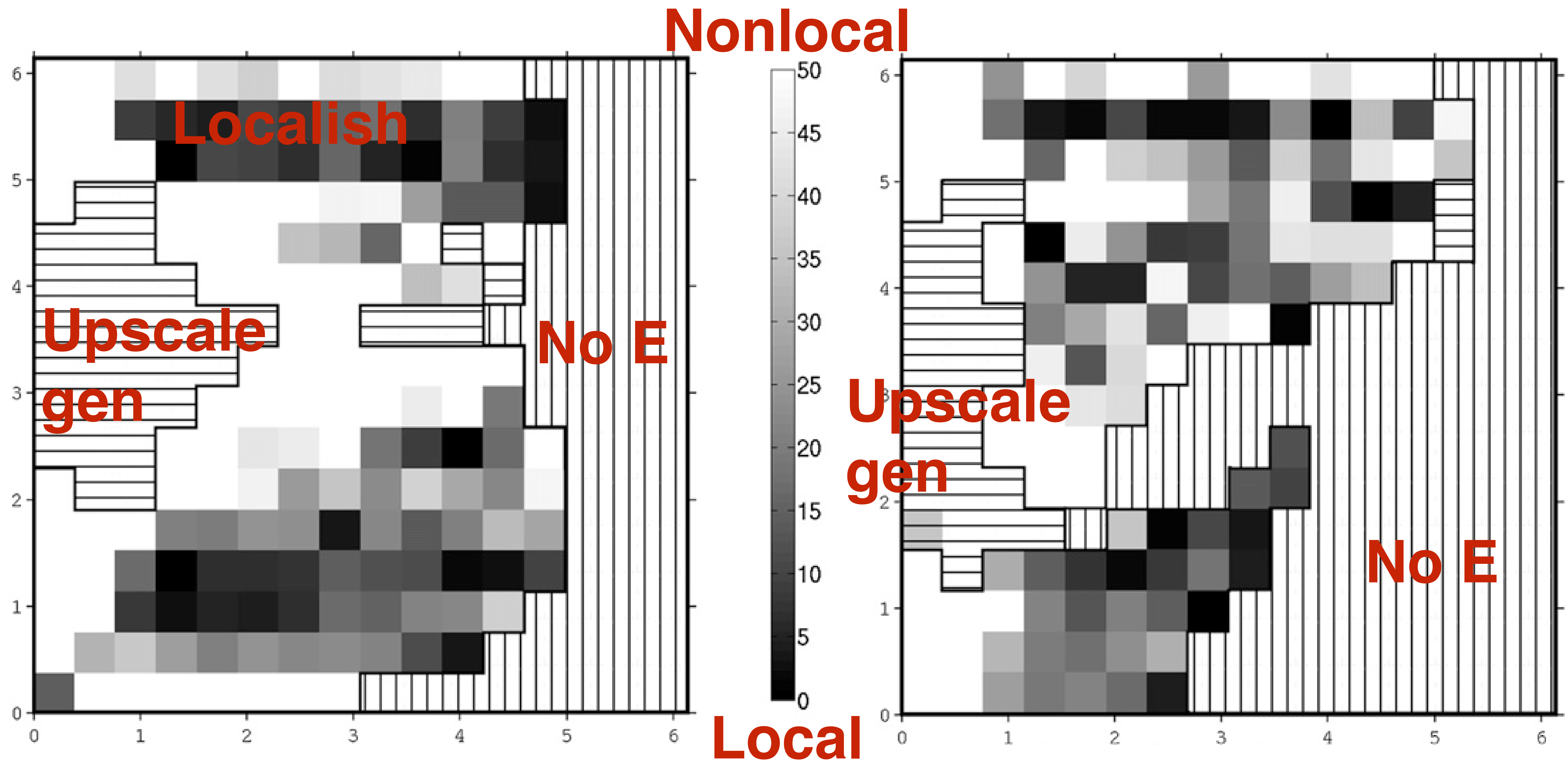
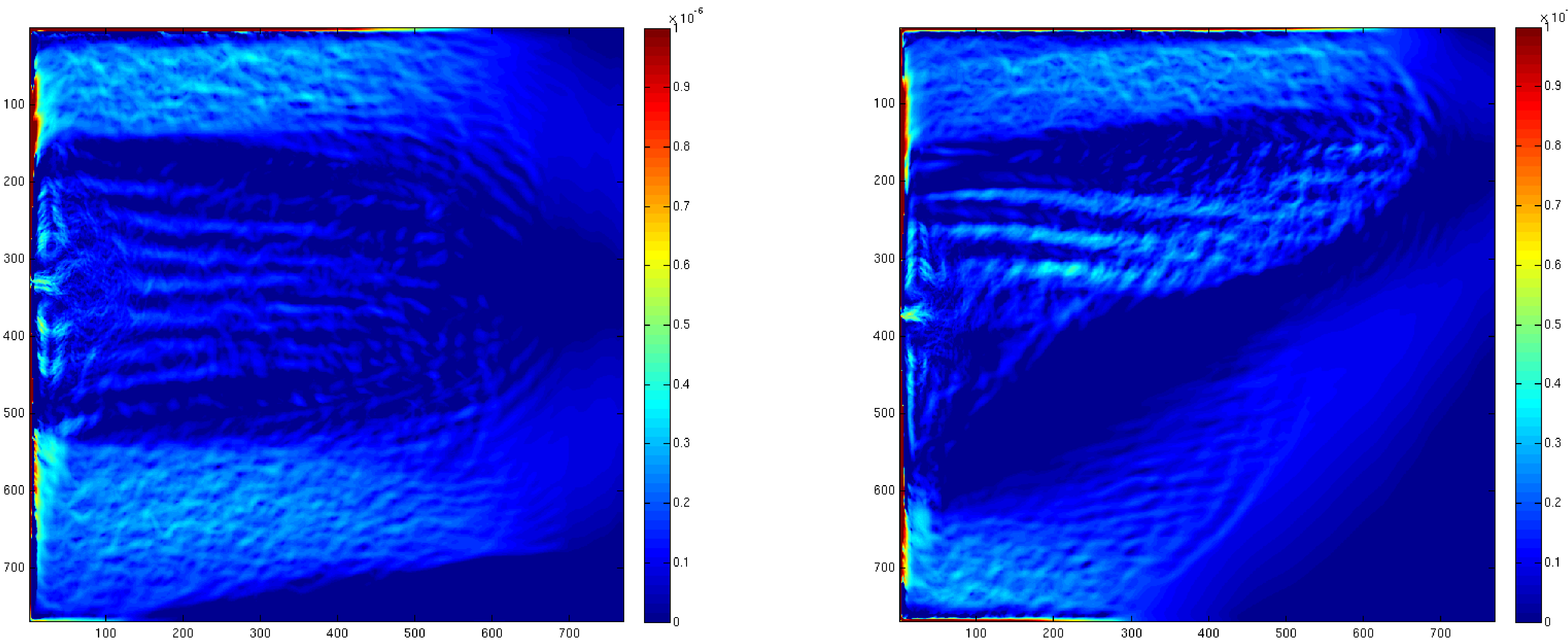


FIG. 5. Basin nonlocality. The ratio of coarse-grained nonlocal terms $\langle F \rangle_t$ to local dissipation $\langle D \rangle_t$ shown as a percentage for the (left) zonal-wind and (right) nonzonal-wind cases. Regions where the nonlinear energy transfer is upscale (negative generation $\langle G \rangle_t$) are indicated by horizontal lines. Regions where the coarse-grained energy density $\langle \tilde{E} \rangle_t$ is less than $10^3\,\mathrm{kg\,s^{-2}}$ are indicated by vertical lines. Coarse-graining is performed by averaging over 16×16 square regions of width 384 km, and $\langle \cdot \rangle_t$ denotes a 10-yr average.

Linear stability of time-mean



Growth rate of baroclinic instability based on large-scale mean shear and PV gradient.

As conjectured by VVS, regions of fast growth rates are localish…

Comparison to periodic runs

There is a plausible correlation between the performance of periodic simulations and the strength of instability.

Our doubly-periodic simulations are most accurate in the regions of local energetics, which are also regions of strong instability.

Our doubly-periodic simulations are least accurate where the energetics are nonlocal, **and where the mean shear is strongly meridional.**

Our results demonstrate that doubly-periodic simulations with meridional shear do not approximate mid-ocean flows with meridional shear.

Conclusions

- Eddy energy budget is nonlocal over a wide region of our idealized model.
- Mesoscale eddy parameterizations might benefit from prognostic models of eddy energy.
- Doubly-periodic simulations do not accurately model regions with nonlocal energetics or nonzonal shear, e.g. on the eastern side of wind-driven gyres. This might be improved by using a time-dependent large-scale flow, instead of the time-mean large-scale flow (e.g. Poulin 2010; Poulin et al. 2010).
- Nonlocal energetics and weak instability are correlated in our simulations, providing support for the conjecture of Venaille et al. (2011) that regions with weak instability are susceptible to nonlocal effects.