A new framework for climate sensitivity and prediction

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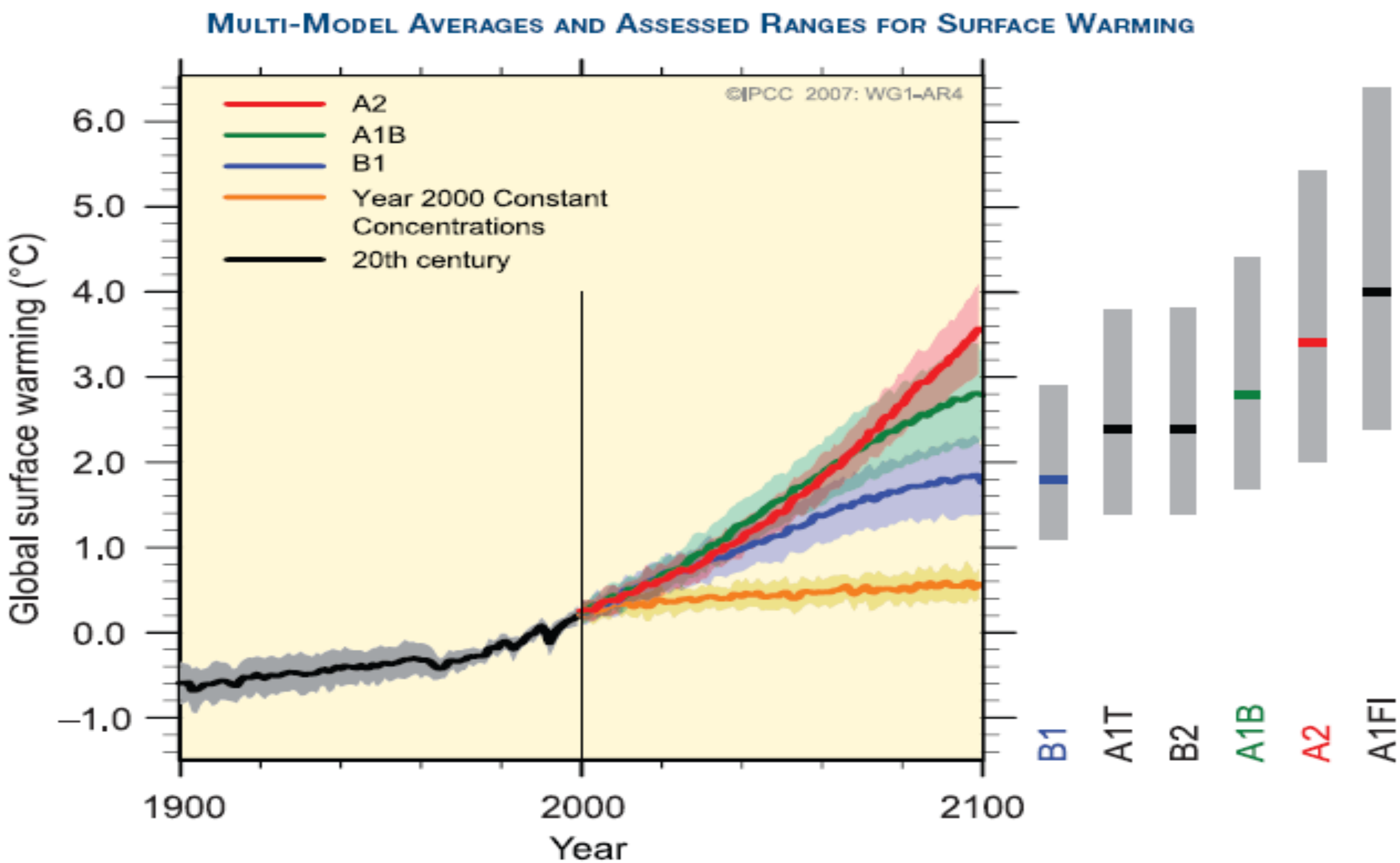

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Introduction: response theory

- Response theory in general is a formalism aimed at describing changes in the statistical properties of a system a under the application of a forcing in terms of the statistical properties of the unperturbed system;

- For **conservative** systems: classical results of equilibrium statistical mechanics, Fluctuation-Dissipation Theorem (FDT);

- For **dissipative** systems: FDT in general does not hold;

- Ruelle (1998 and others) has demonstrated that for a specific class of dynamical systems (Axiom A) it is possible to build a response theory for deviations from non-equilibrium steady states (NESS) formally similar to the equilibrium case;

- Axiom A systems are very specific; applications to more general systems are justified by the Chaotic Hypothesis by Gallavotti and Cohen (1995, 1996): systems with many degrees of freedom can be treated as Axiom A as long as macroscopic observables are considered.

Ruelle response theory (RRT)

- Let us consider a dynamical system for which we assume we can apply RRT, described by the evolution equation

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{X}(\boldsymbol{x})\, f(t)$$

where ***F(x)*** represents the unperturbed dynamics, ***X(x)*** is the structure of the forcing in the phase space, and *f(t)* the time modulation.

- Considering a generic observable Φ, we can write its expectation value as a perturbative expansion

$$\langle \Phi \rangle_f (t) = \langle \Phi \rangle_0 + \sum_{n=1}^{+\infty} \langle \Phi \rangle_f^{(n)} (t)$$

- Each term of the serie can be computed knowing the n-th order **Green function**

$$\langle \Phi \rangle_f^{(n)} (t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} d\sigma_1\, d\sigma_2 \ldots d\sigma_n G_\Phi^{(n)}(\sigma_1, \sigma_2, \ldots, \sigma_n) f(t-\sigma_1) f(t-\sigma_2) \ldots f(t-\sigma_n)$$

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Linear response theory

- Limiting the attention to the linear term

$$\left\langle \Phi \right\rangle_f^{(1)}(t) = \int_{-\infty}^{+\infty} d\sigma_1 G_\Phi^{(1)}(\sigma_1) f(t-\sigma_1)$$

The Green function can be computed knowing the SRB measure of the system;

- It is general a causal function ($G_\Phi^{(1)}(t) = 0$ for $t < 0$). Taking the Fourier transform

$$\left\langle \hat{\Phi} \right\rangle_f^{(1)}(\omega) = \chi_\Phi^{(1)}(\omega) \hat{f}(\omega)$$

where the linear **susceptibility** $\chi_\Phi^{(1)}(\omega)$ is the Fourier transform of $G_\Phi^{(1)}(t)$.

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Kramers-Kronig relations

- The real and imaginary parts of $\chi_{\Phi}^{(1)}(\omega)$ describe the in- and out-of phase response of the system at each frequency (time-scale), and obey the **Kramers-Kronig** relations (KK)

$$\begin{cases} \mathrm{Re}\left[\chi_{\Phi}^{(1)}(\omega)\right] = \dfrac{2}{\pi} P\displaystyle\int_{0}^{+\infty} \dfrac{\omega' \,\mathrm{Im}[\chi_{\Phi}^{(1)}(\omega')]}{\omega'^2 - \omega^2} d\omega' \\ \mathrm{Im}\left[\chi_{\Phi}^{(1)}(\omega)\right] = -\dfrac{2\omega}{\pi} P\displaystyle\int_{0}^{+\infty} \dfrac{\mathrm{Re}[\chi_{\Phi}^{(1)}(\omega')]}{\omega'^2 - \omega^2} d\omega' \end{cases}$$

- Self-consistency relations, have to be satisfied by any linear causal model;

- All this for the linear term; we also have nonlinear susceptibilities for the higher order terms, and related generalized KK relations.

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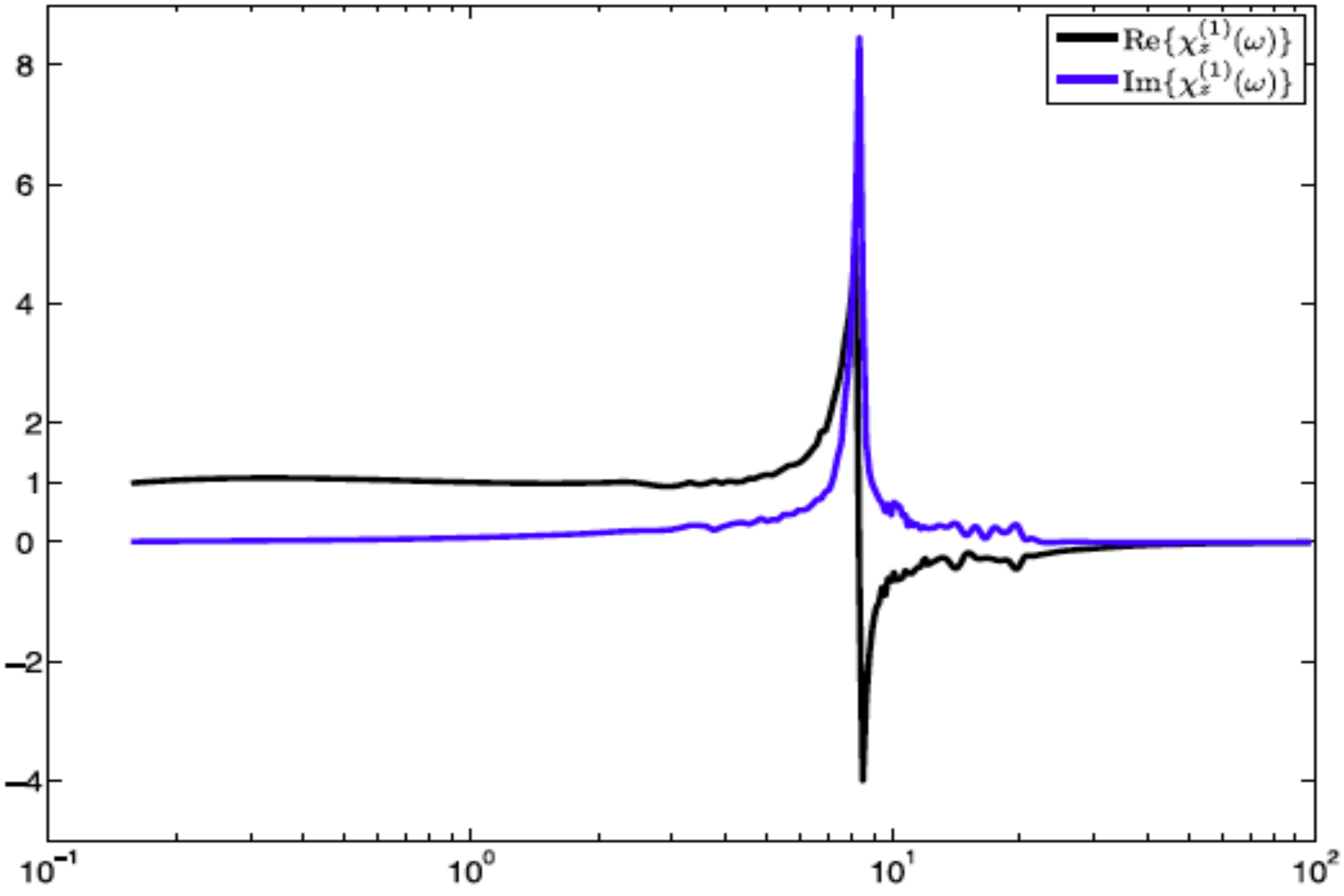
Example: Lorenz 63 model



Lucarini (2009)

Application to climate change experiments

- Let us consider as our dynamical system the climate system as described by a general circulation model (GCM);

- Climate change experiments (IPCC-like): for each emission scenario we change the time modulation of the radiative forcing *f(t),* and we perform a simulation;

- If we know the Green function of an observable, we could in principle avoid doing running the model for every scenario, using simply

$$\left\langle \Phi \right\rangle_f^{(1)}(t) = \int_{-\infty}^{+\infty} d\sigma_1 G_\Phi^{(1)}(\sigma_1) f(t-\sigma_1)$$

- Moreover, the analysis of the susceptibility could tell something on the properties of the observable;

- How do we compute numerically the Green function?

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Application to climate change experiments

- We perform an experiment with a test forcing *f(t)*, and we obtain the susceptibility inverting the equation

$$\left\langle \hat{\Phi} \right\rangle_f^{(1)}(\omega) = \chi_\Phi^{(1)}(\omega)\hat{f}(\omega) \quad \rightarrow \quad \chi_\Phi^{(1)}(\omega) = \frac{\left\langle \hat{\Phi} \right\rangle_f^{(1)}(\omega)}{\hat{f}(\omega)}$$

- From this we can compute the Green function taking the inverse Fourier transform;

- Now we can compute the response to any other forcing *g(t)* using the response formula without running again the GCM

- A good forcing to compute things in this way is the one given by a step-function. In this case

$$f(t) = H(t) \rightarrow \hat{f}(\omega) = \left(\frac{\pi}{2}\delta(\omega) + \frac{i}{\omega}\right) \rightarrow \chi_\Phi^{(1)}(\omega) = -i\omega\left\langle \hat{\Phi} \right\rangle_f^{(1)}(\omega)$$

that is equivalent to

$$G_\Phi^{(1)}(\omega) = \frac{d}{dt}\left\langle \Phi \right\rangle_f^{(1)}(t)$$

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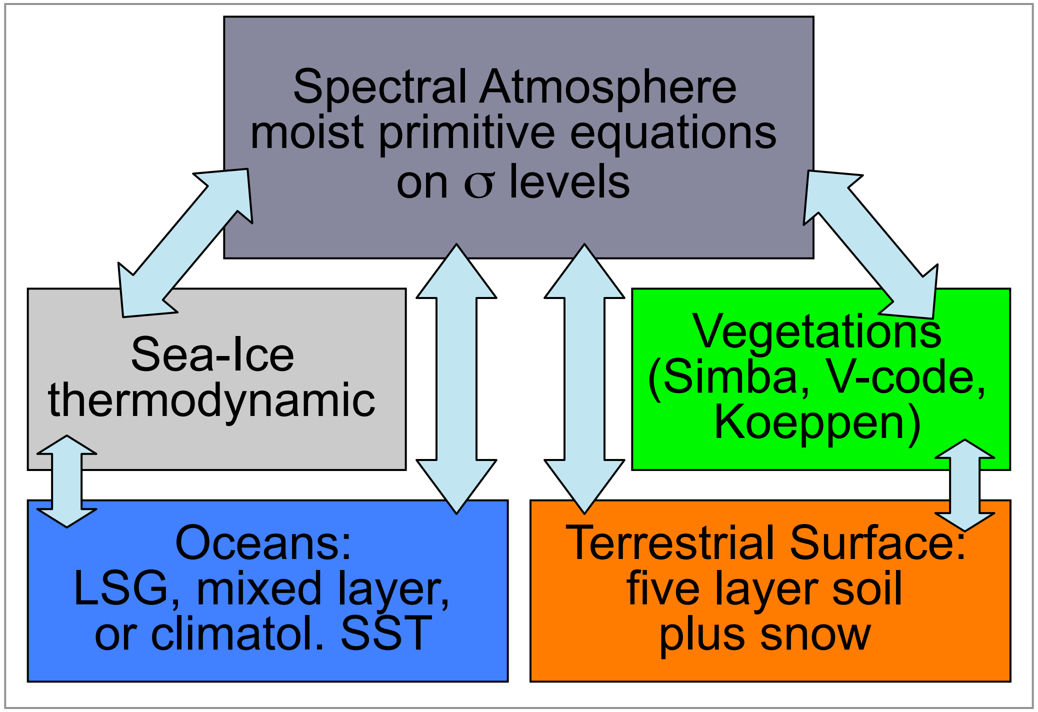
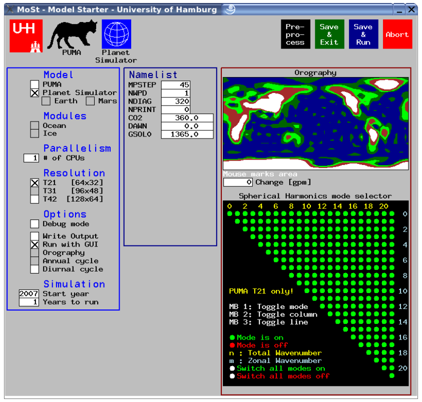
Planet Simulator

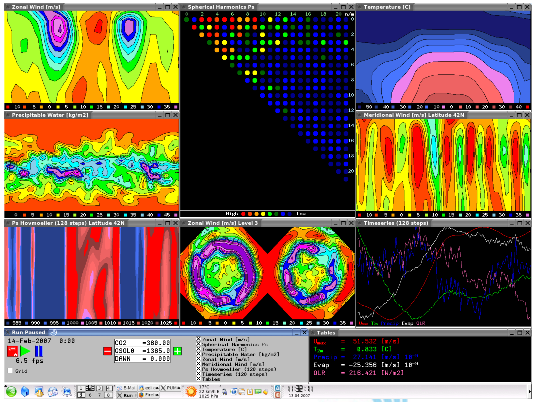


Key features

- portable
- fast
- open source
- parallel
- modular
- easy to use
- documented
- compatible

Model Starter
and
Graphic User Interface





Results: response to [CO_2] doubling and Green function

- Observable: global SST

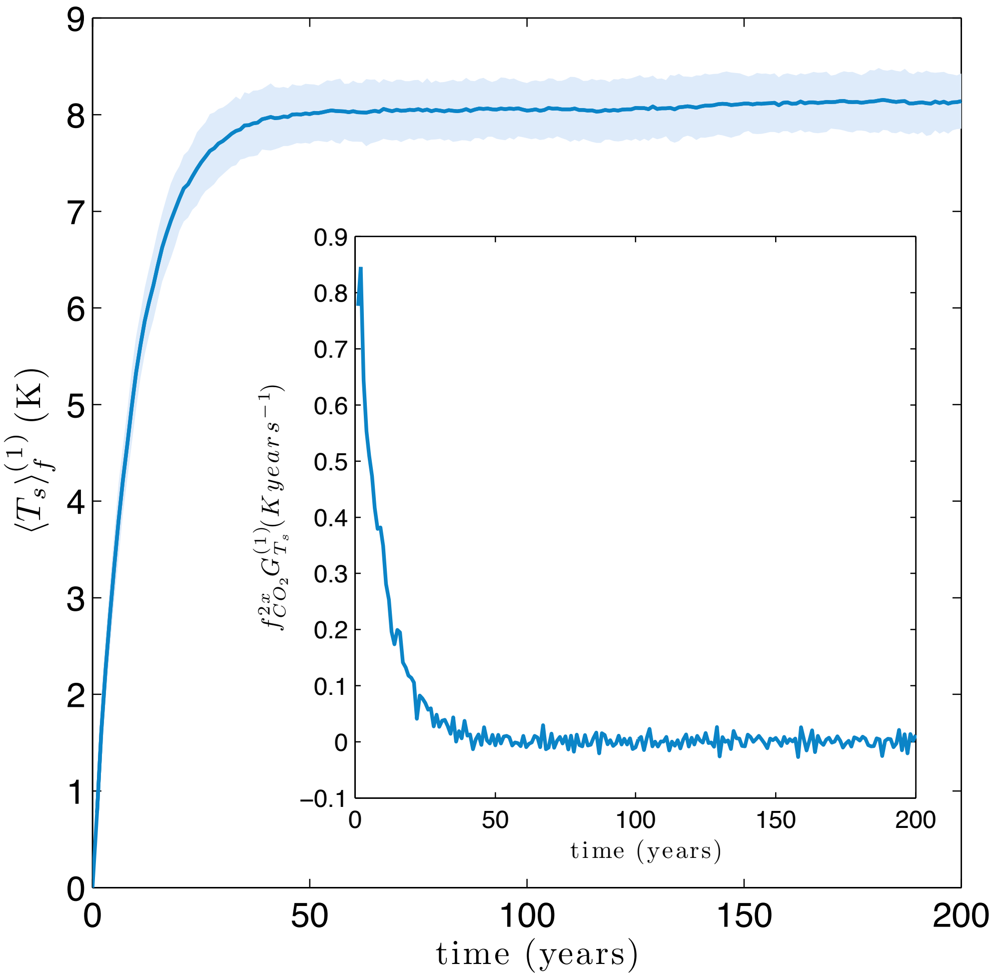
$$\Phi = T_s$$

- Forcing: instantaneous [CO_2] doubling from 360 to 720 ppm

$$f(t) = f_{CO_2}^{2x} H(t)$$

- Ensemble of 200 simulations with different initial conditions, each 200 years long;

- Then we compute the Green function b differentiating the time series of the ensemble average $<T_s>$.



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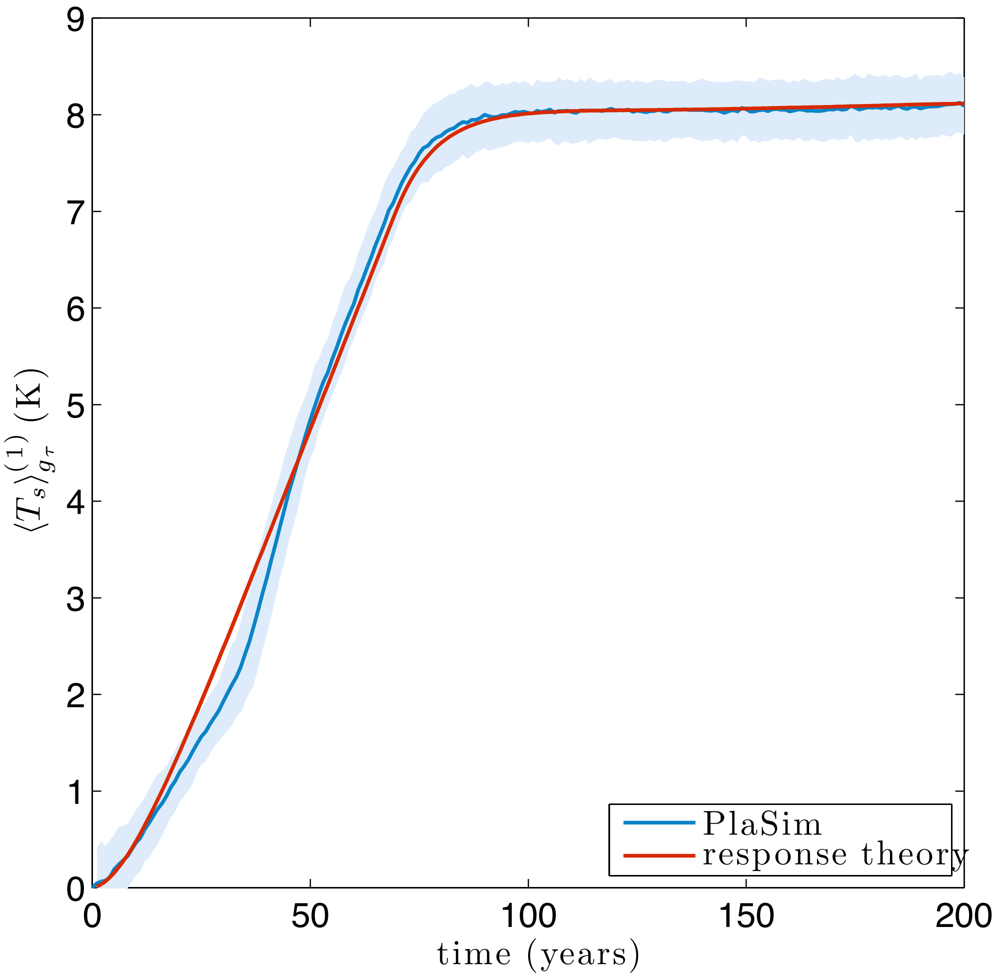
Results: test of prediction with 1% per year [CO_2] increase

- We take another emission scenario: 1% [CO_2] increase per year from 360 to 720 ppm, then constant;

- Forcing *g(t)* is a linear function for the first $\tau = 70$ years and then constant

$$\begin{cases} g(t) = f_{CO_2}^{2x} \dfrac{t}{\tau}, & t < \tau \\ g(t) = f_{CO_2}^{2x}, & t \geq \tau \end{cases}$$

- Prediction by RRT coincides almost perfectly with Plasim simulations.



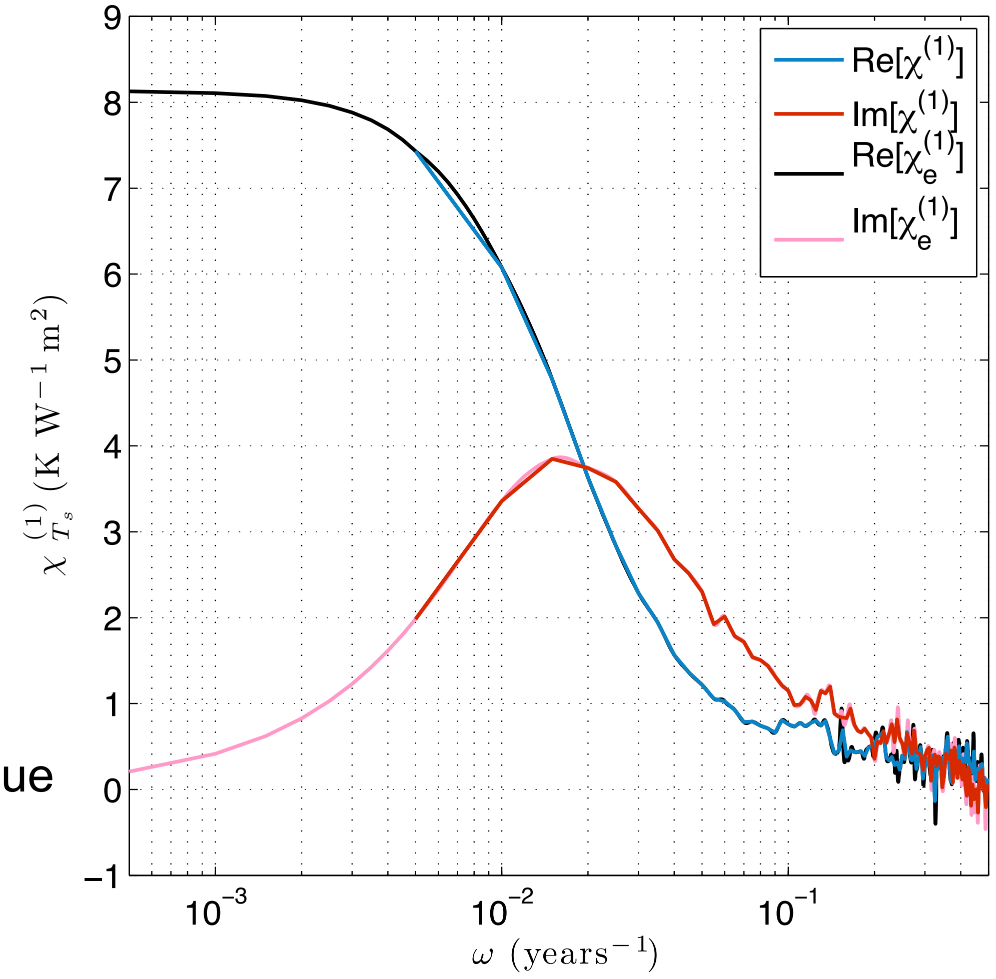
Results: susceptibility of global SST to $[CO_2]$ forcing

- Susceptibility similar to exponential relaxation process, but tails different: complex nature of climate response at multiannual time-scales;

- Value at frequency 0 equivalent to long-term stabilization value of SST increase: Equilibrium Climate Sens.:

$$ECS = f_{CO_2}^{2x} \chi_{T_s}^{(1)}(0)$$

- Therefore when we compute ECS we are computing one (and only one) value of the susceptibility.



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Results: climate sensitivity in response theory framework

- Computing the whole function with RRT is useful because the KK in zero give

$$ECS = f_{CO_2}^{2x}\chi_{T_s}^{(1)}(0) = \frac{2}{\pi}P\int_0^{+\infty}\frac{\mathrm{Im}[\chi_{T_s}^{(1)}(\omega)]}{\omega}d\omega = \frac{2}{\pi}P\int_0^{+\infty}\mathrm{Re}[\langle T_s\rangle^{(1)}(\omega)]d\omega$$

One can check which is the contribution of each frequency (time-scale, therefore physical processes) in determining ECS;

- Intercomparison: by comparing the integrand for two models with a different ECS one can see which are the time-scales mostly responsible for the discrepancy;

- Another measure of CS is the Transient Climate Response (TCR), the temperature increase after a 1% increase of the CO_2, at the moment of doubling $\tau \approx 70$ years. One can show that

$$TCR = ECS - P\int_{-\infty}^{+\infty} f_{CO_2}^{2x}\chi_{T_s}^{(1)}(\omega)\frac{1+\mathrm{sinc}(\omega\tau/2)\mathrm{e}^{-i\omega\tau/2}}{2\pi i\omega}d\omega$$

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Not only global quantities!

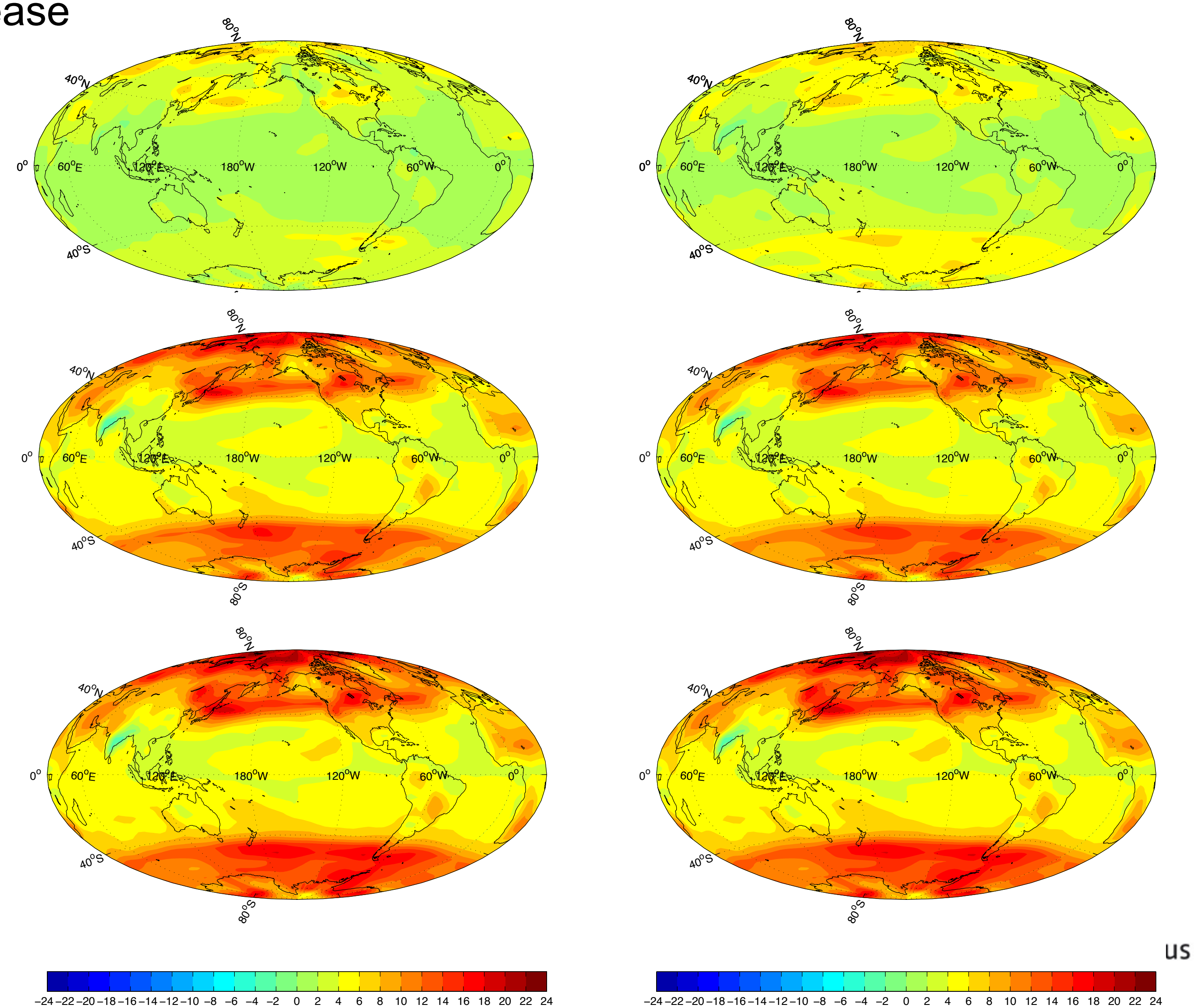
Pattern of SST increase

Plasim | RRT

35 years

70 years

long term



Conclusions

- We have demonstrated the applicability of RRT the analysis of the output of a complex GCM;

- This demonstrates that climate change assessment is a well-defined problem from the mathematical and physical point of view (not obvious!);

- RRT can be used both in a prognostic and diagnostic sense in order to improve climate change studies and optimize the usage of the computational resources;

- In this framework we can approach rigorously the problem of climate sensitivity and climate prediction at specific time scales (for example decadal), both major issues of the last IPCC report.

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Future works

- Studying other variables (water vapor, temperature gradients, radiation fluxes) can give some interesting insights into the properties of the response of the climate system to CO_2 increase;

- Response properties to other forcings (for example, solar forcing) can be interesting for other fields (planetary sciences);

- We can use the theory in order to study systematically simple geophysical models: Lorenz63 (Reick, Lucarini), Lorenz80 (existence of slow manifold, work in progress), others…

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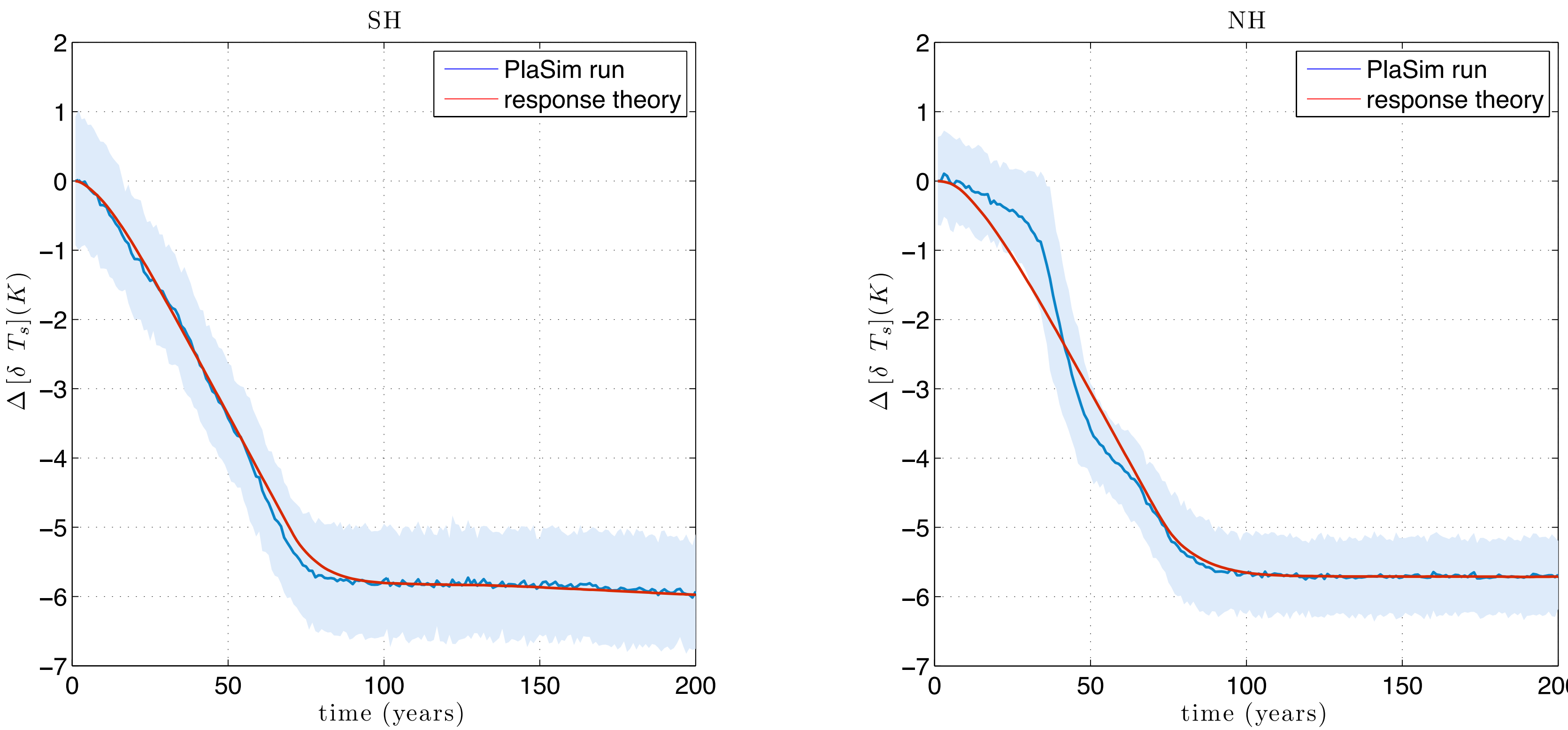
Other quantities: surface temperature gradients



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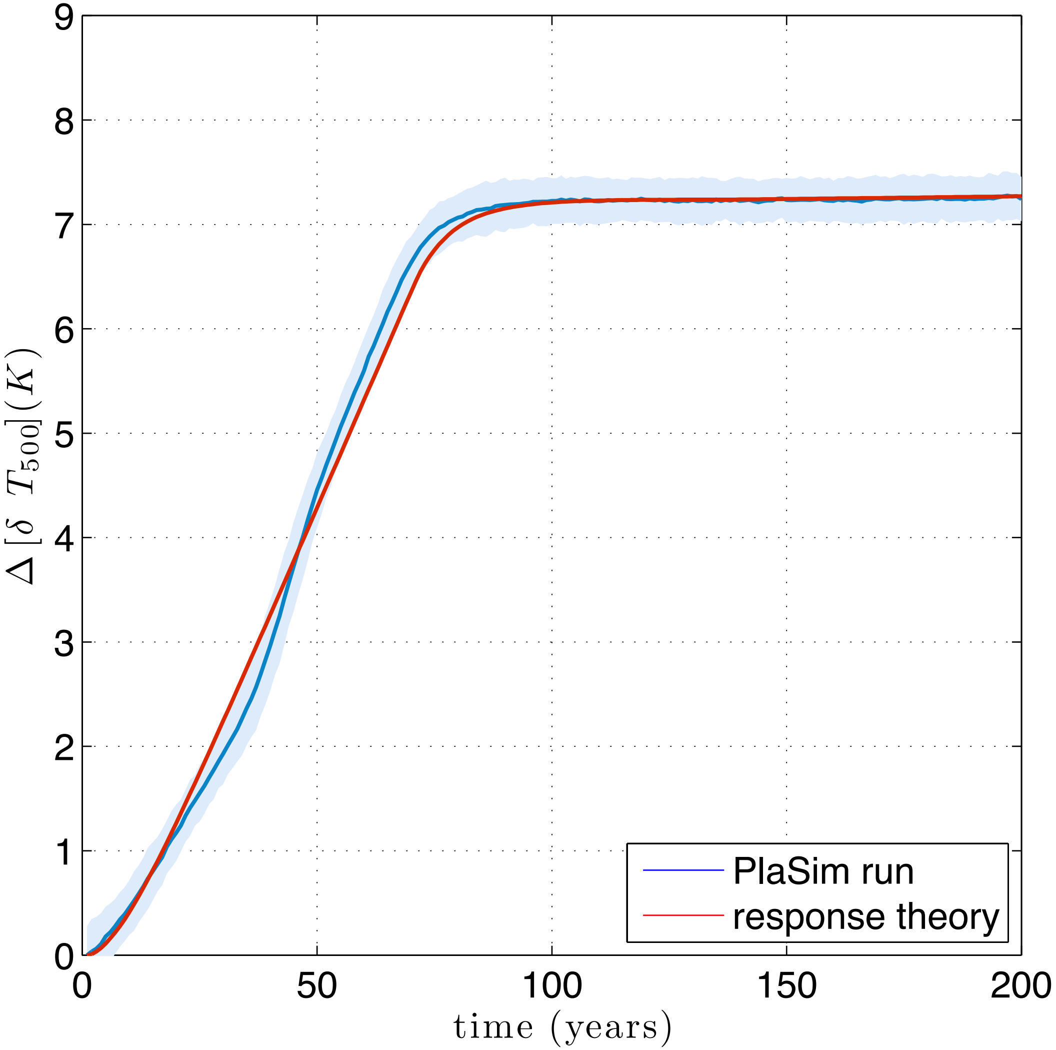
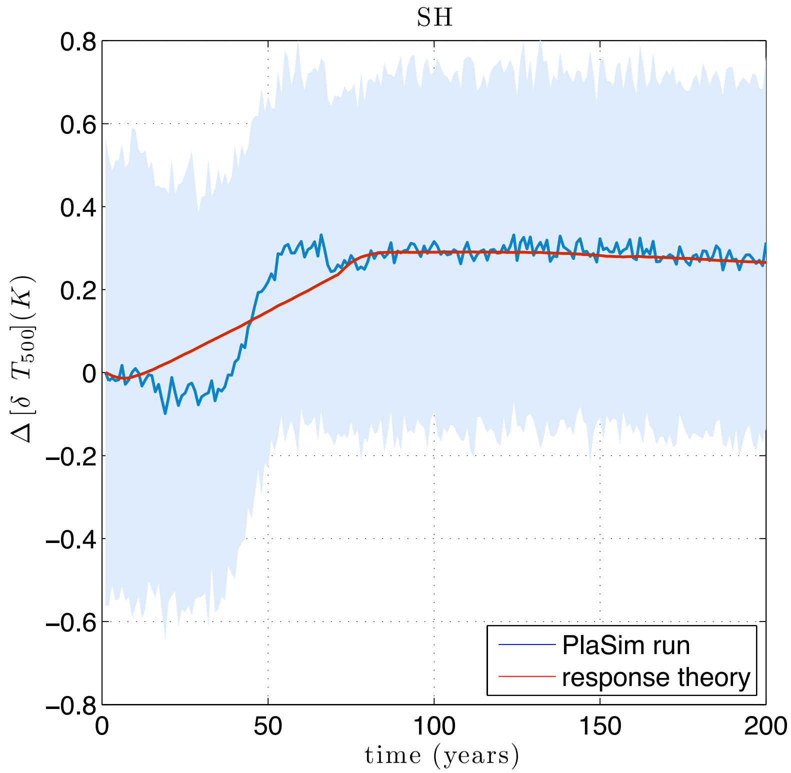
Other quantities: 500 hPa global temperature



Other quantities: 500 hPa temperature gradients



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Other quantities: global vertical (in)stability