A new framework for climate sensitivity and prediction

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1. Summary

- The climate system (CS) is an example of nonequilibrium chaotic dynamical system. Climate change theories and modelling aim at predicting changes in the statistics of climatic observables due to external forcings or internal parameter modulations, such as $[CO_2]$;

- After decades of studies and model development, large uncertainties are still present in the evaluation of the climate sensitivity to $[CO_2]$ increase and in the predictive skills of the general circulation models (GCMs) at specific (e.g. decadal) time-scales;

- Ruelle response theory (RRT) provides a framework to address rigorously the problem of the response for a class of forced-dissipative systems (1). Applications to a system like the CS are justified by the Chaotic Hypotesis (2);

- RRT has been proposed as a tool to build rigorously response operators for generic observables by performing a single forcing experiment exploiting ensemble methods (3).

2. Ruelle linear response theory

Given an Axiom A dynamical system subject to a forcing with structure in the phase-space $\mathbf{X}(\mathbf{x})$ and time modulation $f(t)$

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \mathbf{X}(\mathbf{x})f(t) \tag{1}$$

the deviation of the expectation value of an observable $\langle\Phi\rangle_f(t)$ from a stationary state of the unperturbed dynamics $\langle\Phi\rangle_0$ can be computed as

$$\langle\Phi\rangle_f(t) = \langle\Phi\rangle_0 + \sum_{n=1}^{+\infty}\langle\Phi\rangle_f^{(n)}(t) \tag{2}$$

The linear term can be computed introducing the first order Green function $G_\Phi^{(1)}(t)$ of the observable Φ

$$\langle\Phi\rangle_f^{(1)}(t) = \int_{-\infty}^{+\infty} G_\Phi^{(1)}(\sigma_1)f(t-\sigma_1)d\sigma_1 \tag{3}$$

The Green function is a causal function, $G_\Phi^{(1)}(t) = 0,\ t < 0$. Taking the Fourier transform of (3)

$$\langle\hat{\Phi}\rangle_f^{(1)}(\omega) = \chi_\Phi^{(1)}(\omega)\hat{f}(\omega) \tag{4}$$

where the linear susceptibility $\chi_\Phi^{(1)}(\omega)$ is the Fourier transform of $G_\Phi^{(1)}(t)$. Being $G_\Phi^{(1)}(t)$ a causal function, the real and imaginary parts of $\chi_\Phi^{(1)}(\omega)$ obey the Kramers-Kronig relations (KK)

$$\Re\left\{\chi_\Phi^{(1)}(\omega)\right\} = \frac{2}{\pi}P\int_0^{+\infty}\frac{\omega'\Im\left\{\chi_\Phi^{(1)}(\omega')\right\}}{\omega'^2-\omega^2}d\omega'$$
$$\Im\left\{\chi_\Phi^{(1)}(\omega)\right\} = -\frac{2\omega}{\pi}P\int_0^{+\infty}\frac{\Re\left\{\chi_\Phi^{(1)}(\omega')\right\}}{\omega'^2-\omega^2}d\omega' \tag{5}$$

KK are self-consistency relations linking to each other the in- and out-of phase response of the system at all frequencies.

3. Model and methods

Application to climate response and prediction:

1. we perform with a GCM an ensemble of test forcing experiments increasing $[CO_2]$ with a certain time modulation $f(t)$ starting from different initial conditions on the attractor of the unperturbed system;
2. we compute the ensemble average of the response of an observable $\langle\Phi\rangle_f^{(1)}(t)$, and we compute $\chi_\Phi^{(1)}(\omega) = \langle\hat{\Phi}\rangle_f^{(1)}(\omega)/\hat{f}(\omega)$ from eq. (4);
3. we compute $G_\Phi^{(1)}(t)$ as the inverse Fourier transform of $\chi_\Phi^{(1)}(\omega)$;
4. we consider any different time modulation of the forcing $g(t)$: now the response to the new forcing $\langle\Phi\rangle_g^{(1)}(t)$ can be computed using eq. (3), *without the need to run again the GCM.*

We test this procedure with Plasim, a simplified GCM developed at the University of Hamburg. The atmospheric model includes a full set of physical parameterizations which make it qualitatively comparable with state-of-the art GCMs, and is coupled to a mixed layer ocean. The model is run in T21 horizontal resolution with 10 vertical layers. Daily and seasonal cycles are switched off.

We perform an ensemble of istantaneous $[CO_2]$ doubling experiments starting from 200 different initial conditions taken from a control run with $[CO_2] = 360$ ppm. As observable we consider the global surface temperature (SST) $\Phi = T_s$, and we compute $\chi_{T_s}^{(1)}(\omega)$ and $G_{T_s}^{(1)}(t)$. We perform another ensemble of experiments increasing $[CO_2]$ by 1% per year until it has doubled. By comparing the results of this second ensemble of simulations with the prediction obtained with eq. (3) we test the predictive power and applicability of the theory.

4. Response of global SST

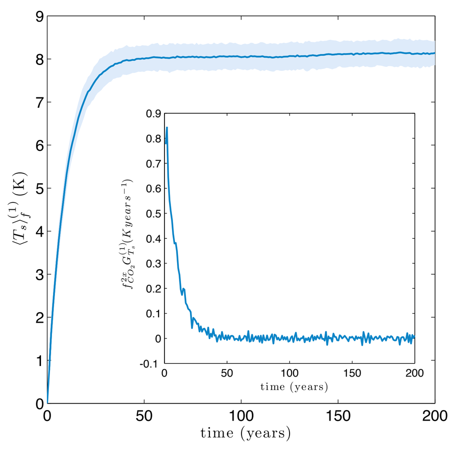


Figure 1

The instantaneous $[CO_2]$ doubling induces a radiative forcing given by a (scaled) Heaviside function $f = f_{2CO_2}^{2x}H(t)$. In this case we can compute directly the (scaled) Green function as

$$f_{2CO_2}^{2x}G_{T_s}^{(1)}(t) = \frac{d}{dt}\langle T_s\rangle_f$$

Figure 1 show $\langle T_s\rangle_f(t)$ and $f_{2CO_2}^{2x}G_{T_s}^{(1)}(t)$ (insert) resulting from the ensemble of simulations.

The 1% $[CO_2]$ increase scenario induces a forcing linear up to $f_{2CO_2}^{2x}$ and constant afterwards. Figure 2 shows in blue the ensemble average of the SST increase from Plasim. The shaded area represents 95% of the ensemble variability. The red curve is the SST increase predicted by RRT computed using equation (3). The agreement between the result of the simulations and the prediction of RRT is remarkable, with only a slight discrepancy during the transient, most probably connected to the activation of the ice-albedo feedback. We stress how, despite the system being highly non linear, the linear theory reproduces the results extremely well.

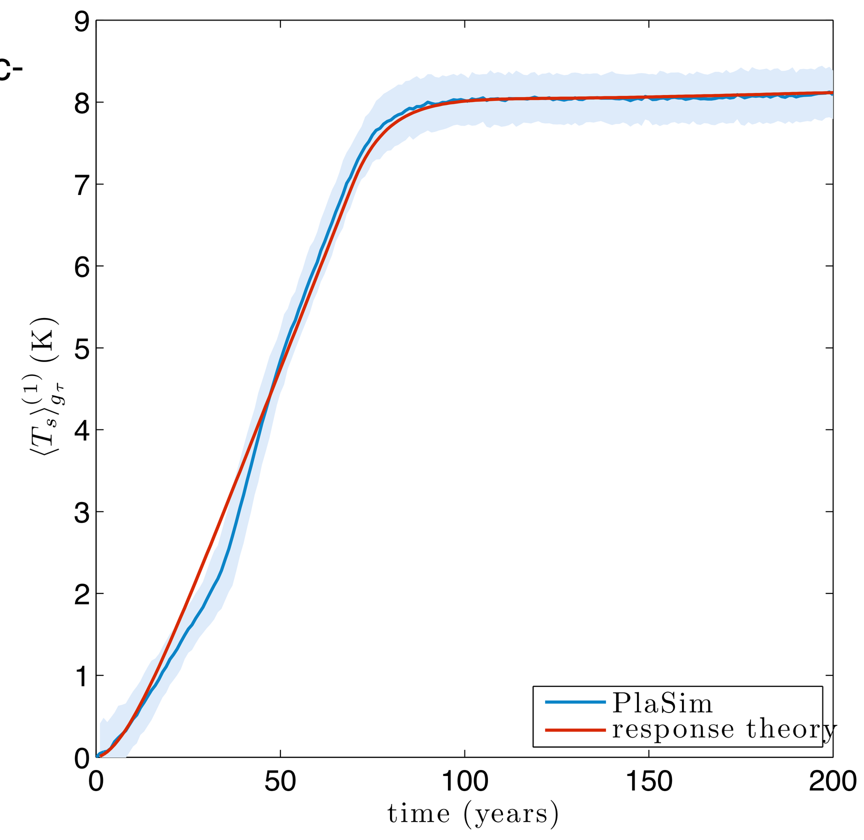


Figure 2

5. Susceptibility of global SST and climate sensitivity

The real and imaginary parts of $f_{2CO_2}^{2x}\chi_{T_s}^{(1)}(\omega)$ are only partially consistent with those of a relaxation process (figure 3). The KK analysis retains most of the irregularity in the high frequency regime (complex behavior of the CS at multiannual scale). The value of $f_{2CO_2}^{2x}\chi_{T_s}^{(1)}(\omega)$ at $\omega = 0$ coincides with the equilibrium climate sensitivity (ECS), the stabilization value of the temperature increase after a $[CO_2]$ doubling. Thanks to the KK

$$ECS = f_{2CO_2}^{2x}\chi_{T_s}^{(1)}(0) = \frac{2}{\pi}P\int_0^{+\infty}\frac{\Im\left\{f_{2CO_2}^{2x}\chi_{T_s}^{(1)}(\omega)\right\}}{\omega}d\omega$$

It is possible to derive formulas for other measures of climate sensitivity, as the transient climate response (TCR) (see (4)), the temperature increase at time τ when $[CO_2]$ has doubled increasing by 1% per year (ca. 70 years)

$$TCR = ECS - f_{2CO_2}^{2x}P\int_{-\infty}^{+\infty}\chi_{T_s}^{(1)}\frac{1+sinc(\omega\tau/2)e^{-i\omega\tau/2}}{2\pi i\omega}d\omega$$

In general, the linear susceptibility contains all the informations about the response of the system in the linear regime at all frequencies (time-scales).

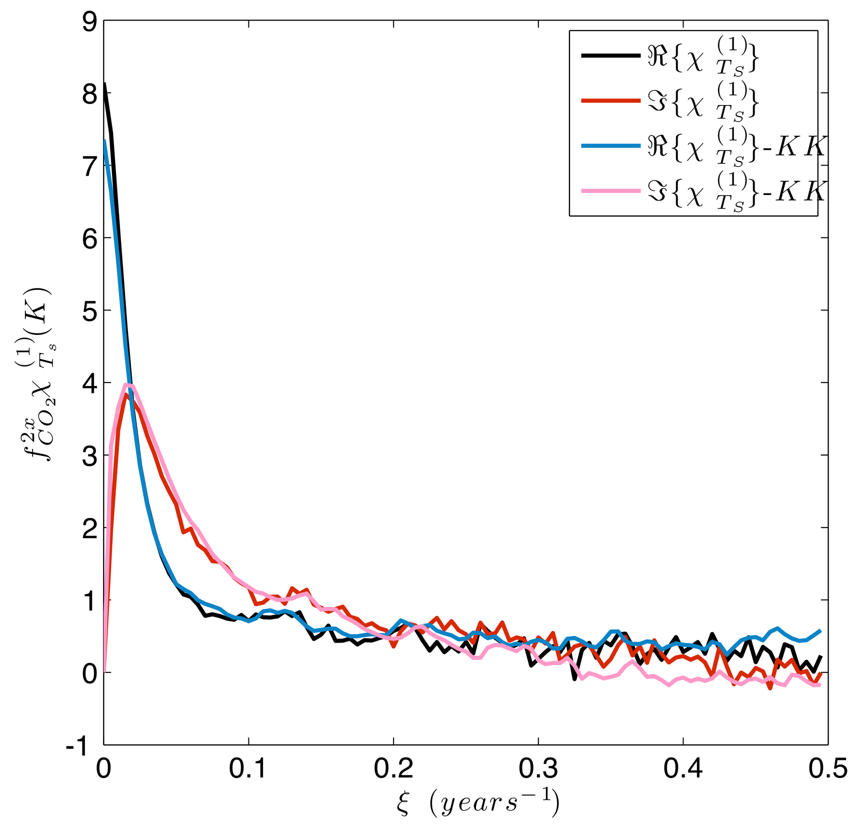


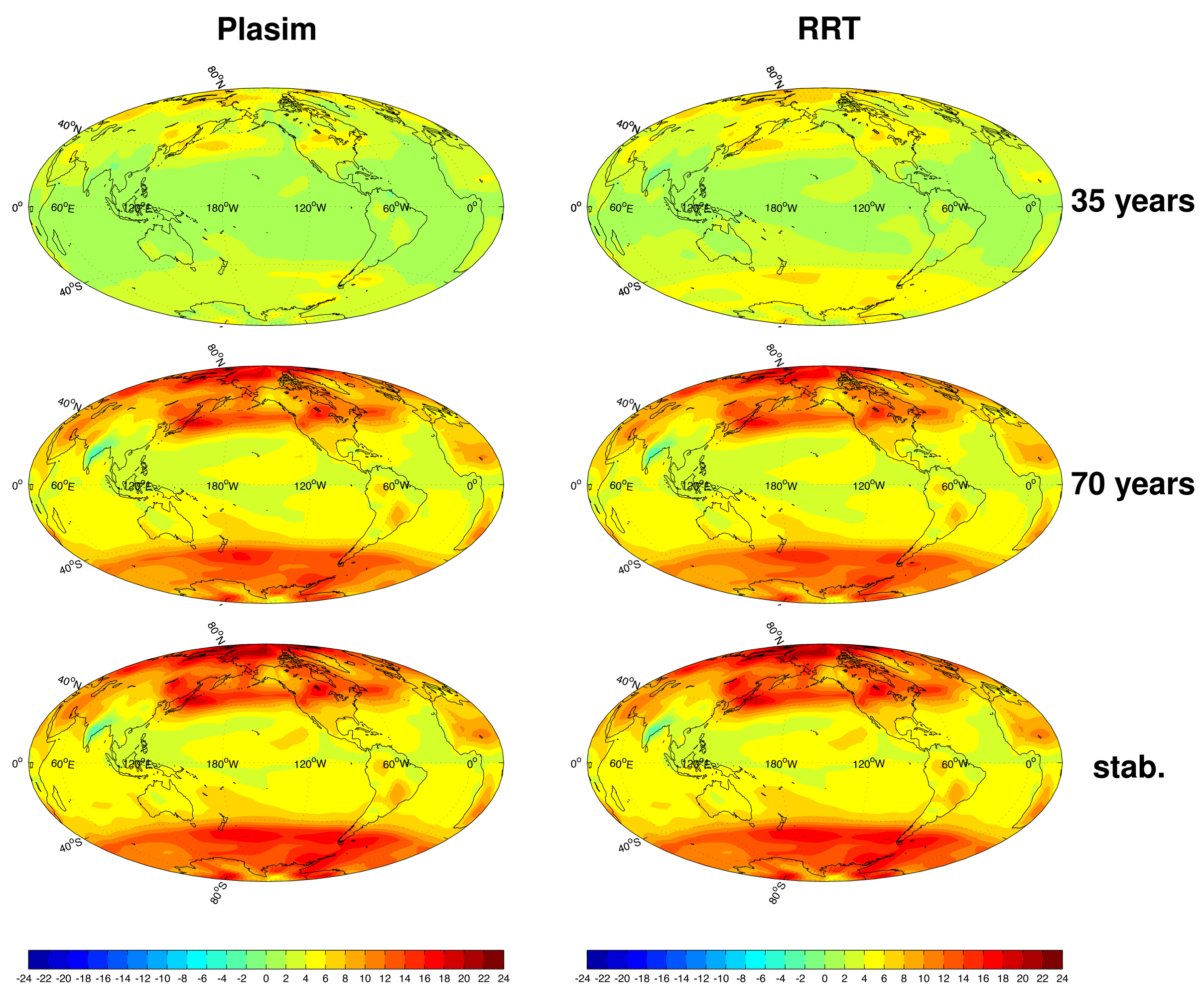
Figure 3

6. Response of SST pattern

In figure 4 we show the results performing the same analysis but considering as observable the local SST at each grid point. This involves computing the Green function of the SST at each grid point. We would expect the skills of RRT in predicting the change in the local SST to be worse than for the global SST. However, it turns out that also locally the prediction is extremely accurate at all times.

The left column shows the patterns of SST increase with 1% per year increase of $[CO_2]$ obtained with Plasim, the right column the ones obtained with RRT. The top row shows the temperature increase at 35 years (in the middle of the transient, where the discrepancy in the global SST between Plasim and RRT is maximum), the middle row at 70 years (the $[CO_2]$ doubling time), the bottom row the long term mean after the stabilization. In all cases the agreement is remarkably good, almost perfect in the last two cases, with some quantitative discrepancy but overall good qualitative agreement also during the transient.

Figure 4



7. Conclusions

- We have shown that linear RRT can be used in order to predict the response of global and local SST to a $[CO_2]$ increase scenario. This implies that climate change assessment is a well defined problem from a mathematical and physical point of view;

- RRT could be used in order to extend the palette of climate change scenario considered by the IPCC without resorting to additional expensive numerical simulations;

- The analysis of the susceptibilities of different models can be used in order to perform a rigorous intercomparison aimed at understanding the origins of persisting discrepancies among models in the different measures of climate sensitivity;

- The analysis of other observables (i.e. precipitable water, vertical and horizontal gradients of temperature) would be of great interest in order to characterize rigorously the response of the CS to climate change;

(1) Ruelle, D., General linear response formula in statistical mechanics, and the fluctuation-dissipation theorem far from equilibrium, Phys. Lett. A, 245, 220224 (1998);
(2) Gallavotti, G.: Chaotic hypotesis: Onsanger reciprocity and fluctuation-dissipation theorem, J. Stat. Phys., 84, 899926, (1996);
(3) Lucarini, V. and Sarno, S. A statistical mechanical approach for the computation of the climatic response to general forcings. Nonlin. Processes Geophys. 18, 728 (2011);
(4) Ragone, F., Lucarini, V., Lunkeit, F. A new framework for climate sensitivity and prediction, Clim. Dyn, submitted.

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