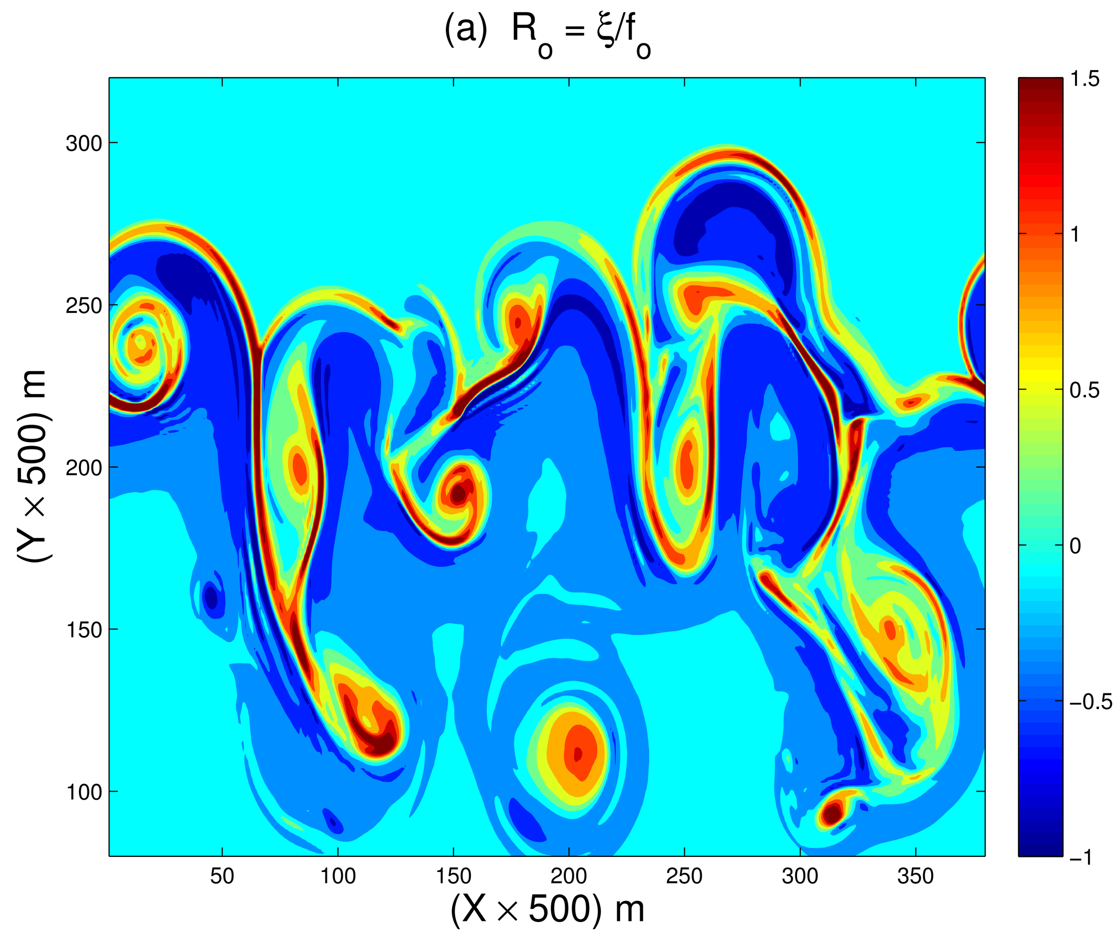
Lagrangian Coherent Structures from 3D Finite Time Lyapunov Exponents

Daniel Mukiibi, Gualtiero Badin, Nuno Serra
Institute of Oceanography, University of Hamburg, Hamburg, Germany.

Introduction

Submesoscale dynamics resulting from the instability of a mixed layer (ML) front generate intensified vertical velocities that are at least an order of magnitude higher than their mesoscale counterparts. The dynamics are also characterised by Rossby numbers, $R_o \geq 1$ [1]. Thus, at these scales the ocean dynamics are relatively 3 dimensional.



The submesoscale dynamics create spatially small scale localised regions (filaments) along which the relative vorticity and vertical velocities are highly intensified. Along these filaments, the vertical stretching of tracer patches becomes important. An extension from 2D to 3D FTLEs in order to identify possible barriers to mixing introduced by this vertical stretching is thus required.

Methodology

The MITgcm is initialised with a ML front overlying an initially motionless pycnocline. The ML front undergoes ageostrophic baroclinic instability. Particles are deployed on a regular grid and are advected with the resultant flow. The positions of the particles are identified for a specific interval of time $(t_2 - t_1)$. The integration is done so as to obtain both forward and backward Finite Time Lyapunov Exponents (FTLEs) [2].

$$\delta(t_2) = \delta(t_1) \mathrm{e}^{\sigma(t_2 - t_1)} \tag{1}$$

$$\delta(t_2) = D\delta(t_1) \tag{2}$$

$$\sigma = \frac{1}{|t_2 - t_1|} \mathrm{In}\left(\sqrt{\lambda_{max}}\right) \tag{3}$$

where D is the three dimension Cauchy-Green deformation tensor. λ_{max} is the maximum of the eigenvalues of the matrix $(D^* D)$. Local extrema of the FTLE field map the Lagrangian Coherent structures (LCS).

References

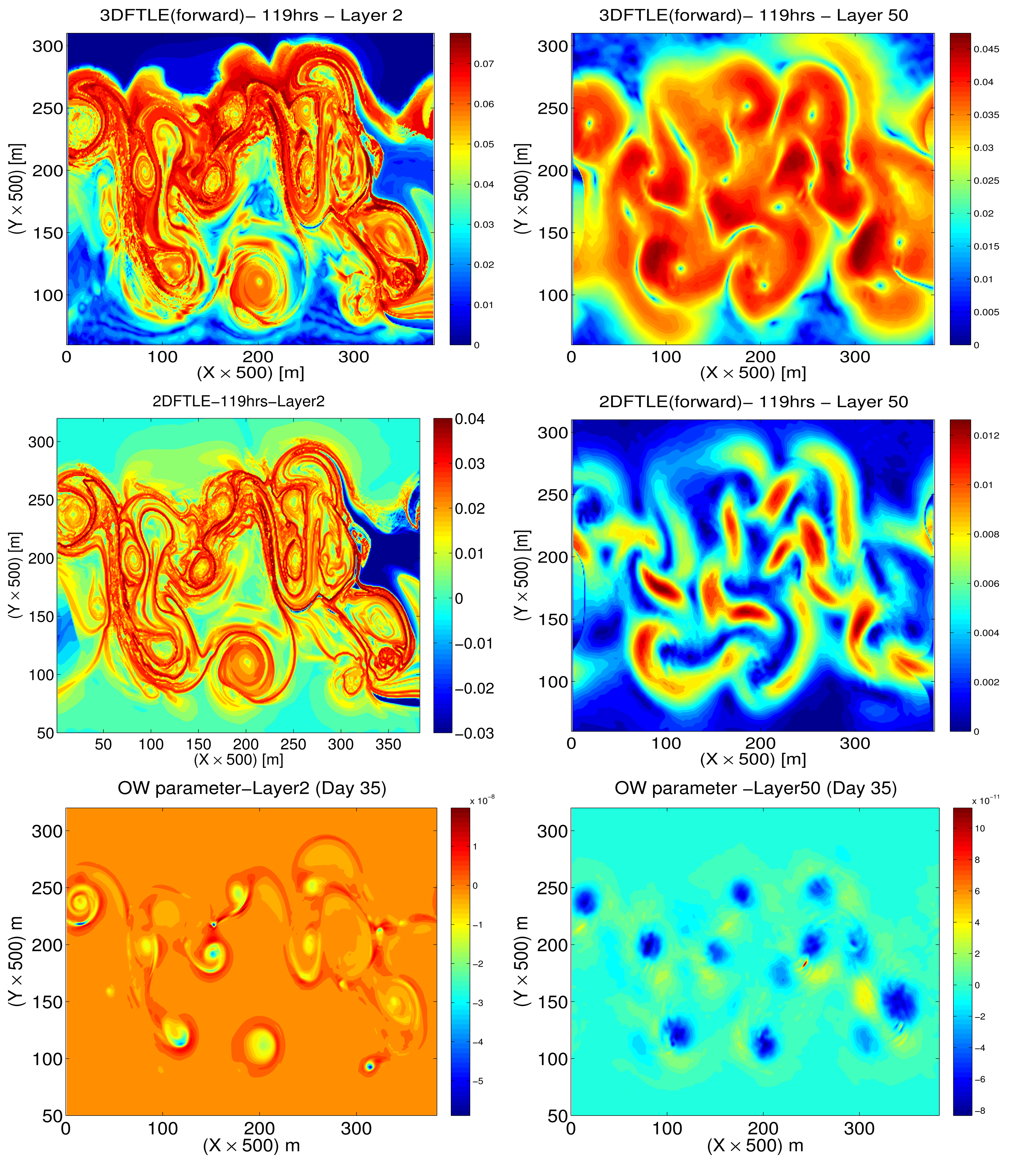
[1] L. N. Thomas, A. Tandon, A. Mahadevani: *Submesoscale processes and dynamics*, Journal of Geophysical Research, 177:17-37, 2008.

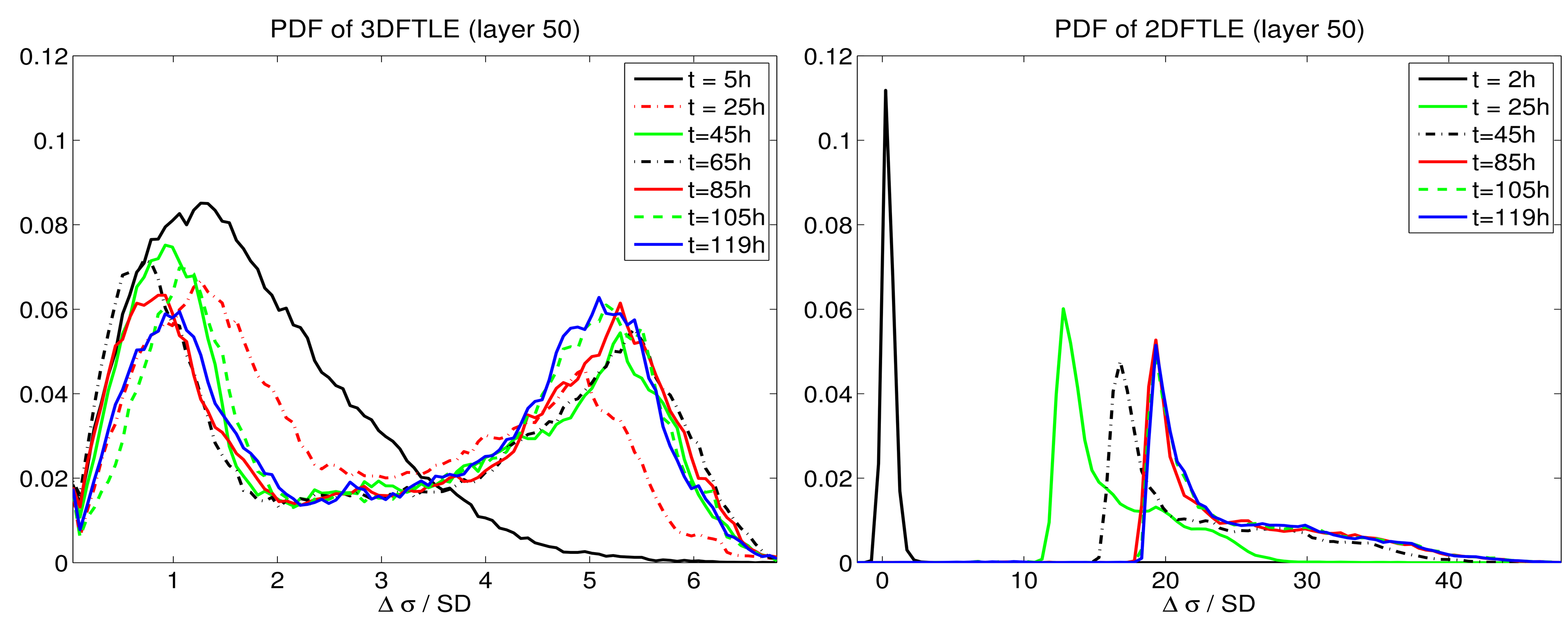
[2] S. C. Shadden, F. Lekien, J. E. Marsden: *Definition and properties of Lagrangian coherent structures from finite-time Lyapunov exponents in two-dimensional aperiodic flows*, Physica D, 212:271-304, 2005.

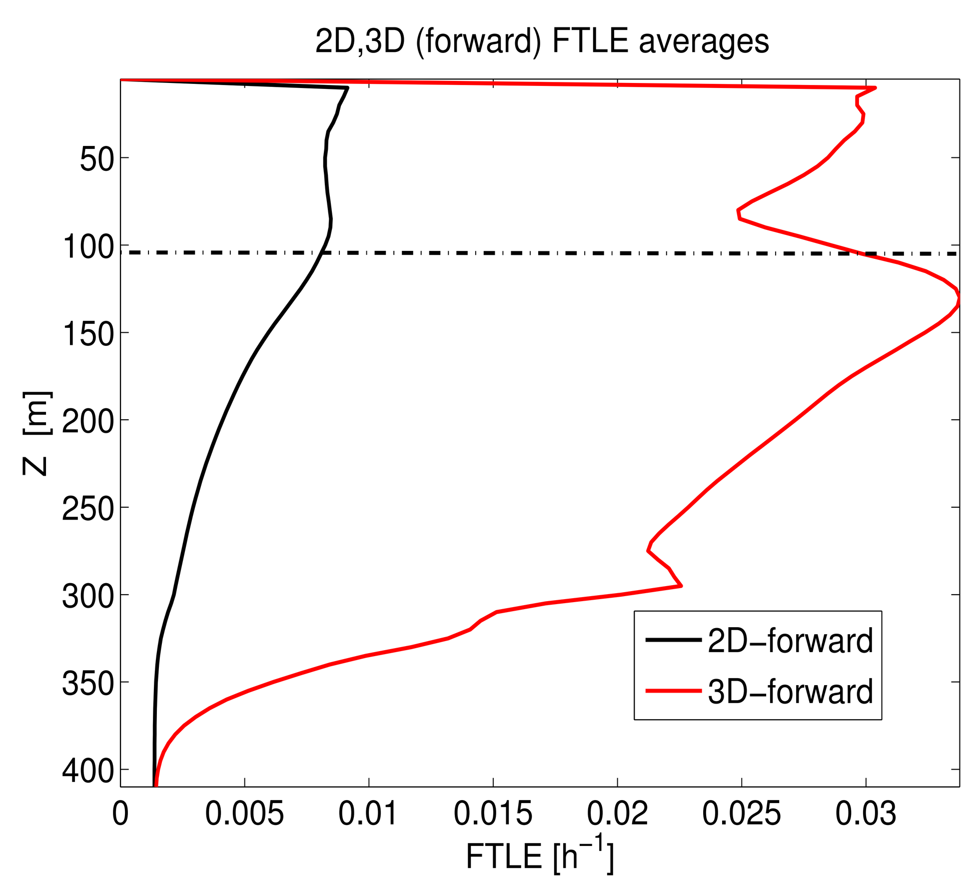
[3] G. Badin, A. Tandon, A. Mahadevani: *Lateral mixing in the pycnocline by Baroclinic Mixed Layer Eddies*, Journal of Physical Oceanography, 41:2080-2101, 2011.

Correspondence: daniel.mukiibi@zmaw.de

Results







- ML instabilities that are generated by mixed layer fronts at sub-mesoscales penetrate into the ocean interior as observed from the high values of FTLEs even in deeper layers [3].
- LCSs computed from from 3D FTLEs reveal more features compared to their 2D counterparts and thus the extension to 3D is deemed more appropriate for obtaining LCSs at submesoscales. Thus, the LCSs in this case are 2D surfaces extending to the ocean interior and can be barriers to horizontal mixing.
- The FTLE technique provides more features about the flow dynamics as compared to the Eulerian Okubo-Weiss parameter which reveals less features.
- The PDFs of 3D FTLEs are observed to be bimodal due to stickiness of certain regions that trap fluid particle trajectories for longer times. These regions may correspond to regions where the stable and unstable manifolds correspond.