A Search for Chaotic Behavior in Stratospheric Variability

Daniela I. V. Domeisen[1] and Gualtiero Badin[1]

[1]Institute of Oceanography, Center for Earth System Research and Sustainability (CEN), University of Hamburg, Hamburg, Germany.

Abstract

Stratospheric extratropical variability in both the Southern and Northern Hemisphere is investigated with respect to chaotic behavior using time series from three different variables (zonal mean zonal wind, temperature and geopotential height) extracted from four different reanalysis products. The time series show red spectra at all frequencies and the probability distribution functions show persistent deviations from a Gaussian distribution, which is found to be due to the transition between summer and winter variability.

To search for the presence of a chaotic attractor the correlation dimension and entropy, the Lyapunov spectrum, and the associated Kaplan–Yorke dimension are estimated. A finite value of the dimensions can be computed for each variable and data product, with a larger spread for the Southern Hemisphere as compared to the Northern Hemisphere.

Reanalysis data

The following reanalysis datasets are used for the analysis:

- **ERAInterim** (Dee et al. 2011):
  - 1 January 1980 to 31 December 2011
  - total of 11 688 daily data points
- **ERA40** (Uppala et al. 2005):
  - 1 January 1958 to 31 December 2001
  - total of 16 071 data points
- **NCEP** (Kalnay et al. 1996):
  - 1 January 1948 to 31 December 2012
  - total of 23742 points
- **NCEP2** (Kanamitsu et al. 2002):
  - 1 January 1979 to 31 December 2012
  - total of 12 419 data points

The seasonal cycle has been removed from each reanalysis time series.

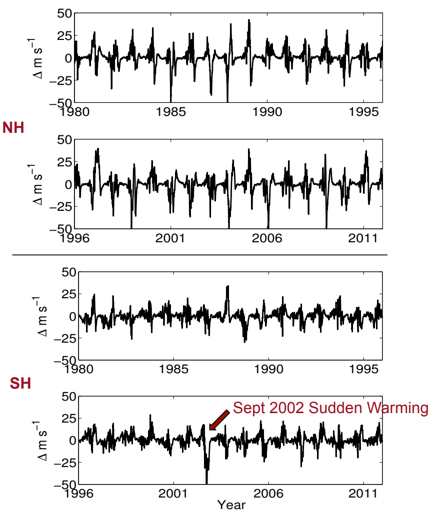

Figure 1: ERAinterim zonal mean zonal wind timeseries with the seasonal cycle removed for (top) the NH and (bottom) the SH.

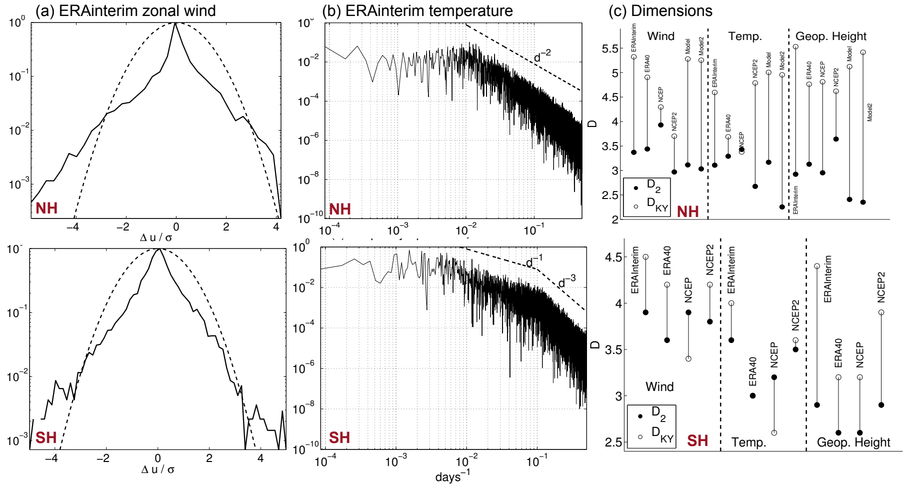

Figure 2: (a) Probability density functions (PDFs) of the zonal mean zonal wind for ERAinterim for the full time series. (b) Frequency spectra for zonal mean zonal wind at 60∘S and 10hPa for ERAinterim for the full time series. (c) Summary of the correlation dimension (D2) and the Kaplan-Yorke dimension (D_{KY}) found in the re-analyses for all three variables.

Theory

The dimension of the attractor is calculated following the theory of Grassberger and Procaccia (1983a,b). The time series are first embedded in a M-dimensional space using a delay time τ. The resulting vectors take the form

$$\vec{x}(t) = \{x(t), x(t+\tau), ..., x[t+(M-1)\tau]\} \quad (1)$$

The maximum embedding dimension is defined following Ruelle (1990) as $M = 2\log_{10}N$, where N is the number of data points in each time series. Given the length of the time series considered in this analysis, the value of M = 8 will be used throughout the study. The delay time τ will instead be evaluated as the first minimum of the mutual information (Fraser and Swinney 1986) or as the time required to the autocorrelation function to reach the value of 1/e. The delay time varies between τ = 34 and τ = 196.

The correlation dimension D_2 is calculated as

$$D_2 = -\frac{\partial \ln C_2(\epsilon)}{\partial \ln \epsilon}$$

where $C_2(m, \varepsilon)$ is the correlation between the points obtained from (1) within a ball of radius ε. For deterministic chaos, D_2 must converge to a value for a large enough interval of ε.

The Lyapunov exponents are calculated following the algorithm of Sano and Sawada (1985). The number of Lyapunov exponents is the same as the dimension of the embedding phase space, while the ordered Lyapunov spectra $\lambda 1 \geq ... \geq \lambda_M$ define the Kaplan-Yorke dimension

$$D_{KY} = j + \frac{1}{|\lambda_{j+1}|}\sum_{i=1}^{j}\lambda_i$$

where the sum is over the first j non-zero $|\lambda_j+1|$ Lyapunov exponents.

The dimensions are related through

$$D_\lambda \leq D_2 \leq D_{KY}$$

where D_λ is the number of positive Lyapunov exponents.

Results

While the existence of a strange attractor for climate has been proven difficult and limited by a number of factors, Lorenz (1991) showed that the dimension of the attractor determined from the analysis of time series depends on the chosen variable, and he proposed that the finite attractor dimensions found by some of the earlier studies might correspond to the dimension of a subsystem. In this study we again raise the question of the existence of a finite attractor, but guided by the studies described above we restrict our attention to a climate subsystem, represented by the NH extratropical stratosphere.

The goal is to study the dependence of the dimension of the attractor on the choice of the variable to be analyzed, using four different reanalyses.

Results show that the analysis of the time series exhibits a convergence to finite dimensions for all the different variables and data sets, though with a large range of results for the same analysis evaluated in different data sets, or for different variables within the same data set.

The different measurements of the dimension of the attractor show a smaller spread for the SH than for the NH.

Note that both in the SH and in the NH the values of D_2 change not only in different data sets but also between variables. The obtained values must be considered with caution, due to the large difference between different dimensions obtained using the same variable, which is a signature of large intermittency in the system. It is therefore not clear if the different dimensions are given by the different results for different variables (Lorenz, 1991) although they are dynamically related, or if it arises from inconsistency between the data sets or methodology. The results obtained in this study question thus the characteristics or the very existence of a strange attractor for stratospheric variability, leaving open the question if the climate system can be modeled as a stochastic system (Hasselmann 1976). Further work is required to determine the correct statistical representation of the variability.

References

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