Stochastic Superparametrization in quasigeostrophic models

Shafer Smith (Courant/NYU)
Ian Grooms (Courant/NYU —> UC Boulder/Math)
Andy Majda (Courant/NYU)

Workshop on Energy Transfer
MPI, Hamburg
20-22 April 2015

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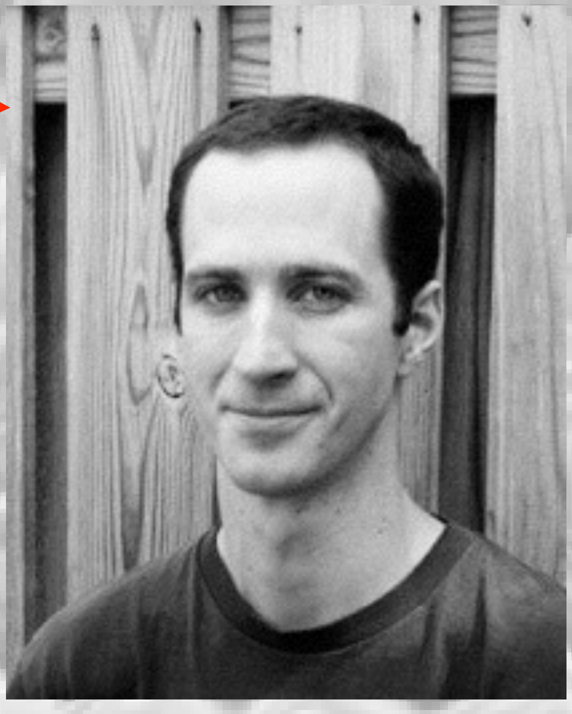

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Superparametrization = multiscale parametrization

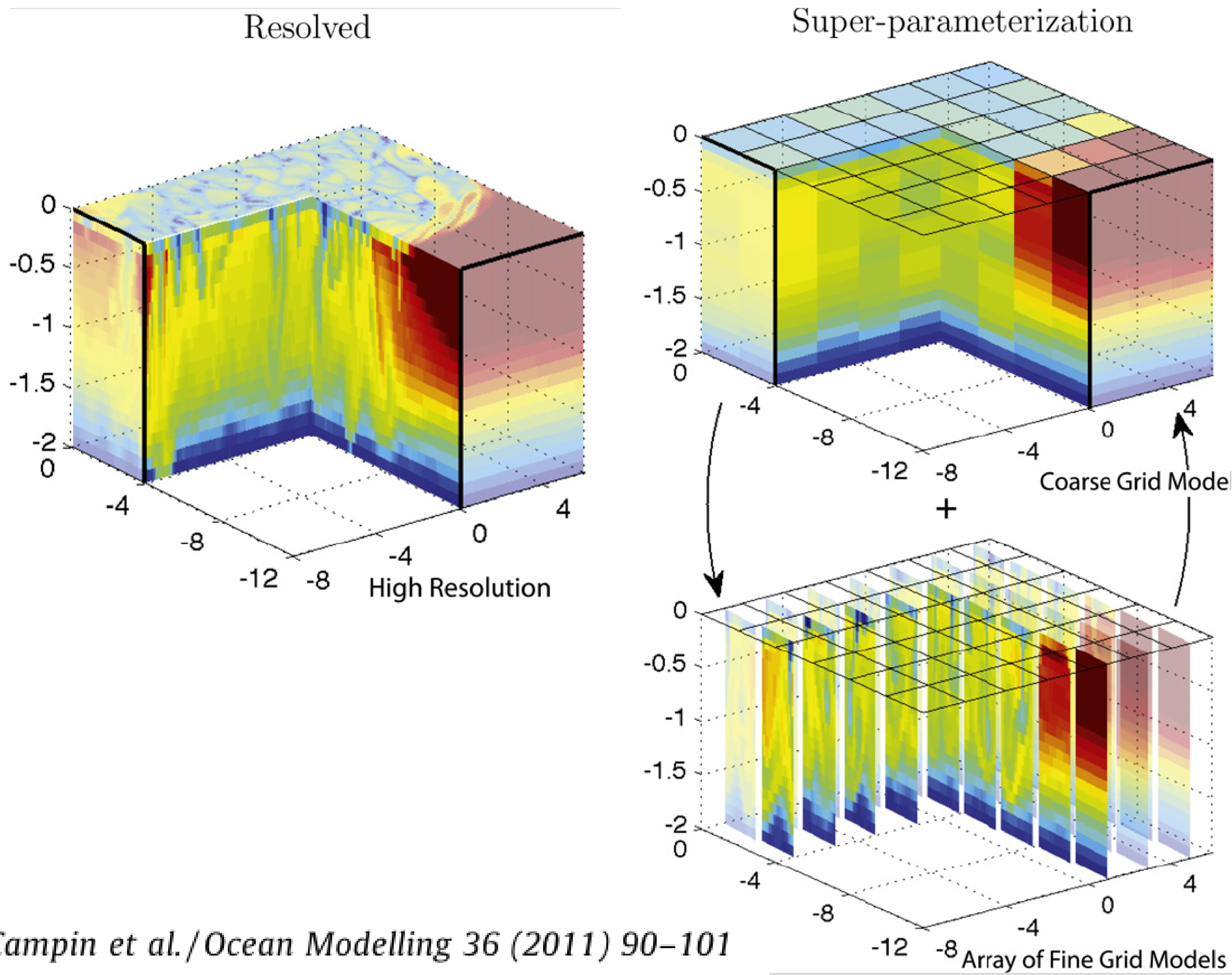
- Superparametrization (SP) developed for tropical moist convection: Randall (CSU), Khairoutdinov (Stonybrook), Arakawa (UCLA), Grabowski (NCAR) — great results there.

- IDEA: Instead of trying to parameterize unresolved dynamics, compute them on each grid cell

- SP acts like a stochastic parameterization because feedback to large-scale is chaotic, not a deterministic function of large scales

- Here, we reduce subgrid model to a stochastic model, and use cheap methods to obtain fluxes to large scale.

Example ocean application: Deep convection



J.-M. Campin et al. / Ocean Modelling 36 (2011) 90–101

This talk

Review idea of SP for 2-layer QG from these papers

Efficient stochastic superparameterization for geophysical turbulence

Ian Grooms and Andrew J. Majda *

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Stochastic superparameterization in quasigeostrophic turbulence

CrossMark

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Show application to 2-layer ACC type QG sim

Stochastic superparameterization in a quasigeostrophic model of the Antarctic Circumpolar Current

CrossMark

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Stochastic SP for 2-layer QG

The setting is quasigeostrophic (QG) dynamics in a two-layer model with equal depths, on a β-plane and forced by imposed baroclinic shear $U \equiv U_1 = -U_2$

$$\partial_t q_1 + \nabla \cdot (\mathbf{u}_1 q_1) + \partial_x q_1 + (k_\beta^2 + k_d^2) v_1 = \nu \nabla^8 q_1$$
$$\partial_t q_2 + \nabla \cdot (\mathbf{u}_2 q_2) - \partial_x q_2 + (k_\beta^2 - k_d^2) v_2 = -r \nabla^2 \psi_2 + \nu \nabla^8 q_2$$

where

$$\mathbf{u}_j = \nabla^\perp \psi_j, \quad q_j = \nabla^2 \psi_j + \frac{k_d^2}{2}(\psi_{3-j} - \psi_i), \quad k_\beta = \sqrt{\frac{\beta}{U}} L$$

with $j = 1, 2$.

Stochastic SP for 2-layer QG

Reynolds averaging* of governing equations gives

$$\partial_t \overline{q}_j = -\nabla \cdot (\overline{\mathbf{u}_j q_j}) + (-1)^j \partial_x \overline{q}_j - \Pi_j \partial_x \overline{\psi}_j - \delta_{j2} r \nabla^2 \overline{\psi}_j - \nu \nabla^8 \overline{q}_j,$$

$$\partial_t q'_j = -\nabla \cdot (\mathbf{u}'_j q'_j)' - (\overline{\mathbf{u}}_j - (-1)^j \hat{\mathbf{x}}) \cdot \nabla q'_j - \mathbf{u}'_j \cdot \nabla \overline{Q}_j - \delta_{j2} r \nabla^2 \psi'_j - \nu \nabla^8 q'_j$$

where $\Pi_j = k_\beta^2 - k_d^2 (-1)^j$ and $\overline{Q}_j = \Pi_j y + \overline{q}_j$ and:

$$\nabla \cdot (\overline{\mathbf{u}_j q_j}) = \nabla \cdot (\overline{\mathbf{u}}_j \overline{q}_j) + \nabla \cdot (\overline{\mathbf{u}'_j q'_j})$$

$$\nabla \cdot (\overline{\mathbf{u}'_j q'_j}) = \frac{k_d^2 (-1)^j}{2} \nabla \cdot (\overline{\mathbf{u}'_j (\psi'_1 - \psi'_2)}) + \left(\partial_x^2 - \partial_y^2\right) \overline{u'_j v'_j} + \partial_{xy} \left(\overline{(v'_j)^2} - \overline{(u'_j)^2} \right)$$

* a low-pass filter is more appropriate but gets to the same place eventually.

Point approximation for eddy equation

Apply ‘point approximation’ to get eddy equations for SP:

1. Eddy variables depend on new coordinates τ, $\tilde{x}$, $\tilde{y}$.
2. Overbar is re-interpreted as an average over the new coordinates.
3. Large-scale variables have no dependence on the new coordinates.

$$\partial_t q_j' = -\nabla \cdot (\mathbf{u}_j' q_j')' - (\overline{\mathbf{u}}_j - (-1)^j \hat{\mathbf{x}}) \cdot \nabla q_j' - \mathbf{u}_j' \cdot \nabla \overline{Q}_j - \delta_{j2} r \nabla^2 \psi_j' - \nu \nabla^8 q_j'$$

$$\partial_\tau q_j' = -\tilde{\nabla} \cdot (\mathbf{u}_j' q_j')' - (\overline{\mathbf{u}}_j - (-1)^j \hat{\mathbf{x}}) \cdot \tilde{\nabla} q_j' - \mathbf{u}_j' \cdot \nabla \overline{Q}_j - \delta_{j2} r \tilde{\nabla}^2 \psi_j' - \nu \tilde{\nabla}^8 q_j'$$

One could run an SP based on these equations, and it would allow baroclinic instability, but it would be too expensive (and probably fail in some situations — ask Ian).

Gaussian eddy closure

'Gaussian Closure' (GC) eddy equations: Replace nonlinear products of eddy variables by Gaussian additive stochastic forcing F_j and deterministic damping $\Gamma q_j'$:

$$\partial_\tau q_j' = -\tilde{\nabla} \cdot (\mathbf{u}_j' q_j')' - (\overline{\mathbf{u}}_j - (-1)^j \hat{\mathbf{x}}) \cdot \tilde{\nabla} q_j' - \mathbf{u}_j' \cdot \nabla \overline{Q}_j - \delta_{j2} r \tilde{\nabla}^2 \psi_j' - \nu \tilde{\nabla}^8 q_j'$$

$$\partial_\tau q_j' = F_j - \Gamma q_j' - (\overline{\mathbf{u}}_j - (-1)^j \hat{\mathbf{x}}) \cdot \tilde{\nabla} q_j' - \mathbf{u}_j' \cdot \nabla \overline{Q}_j - \delta_{j2} r \tilde{\nabla}^2 \psi_j' - \nu \tilde{\nabla}^8 q_j'$$

Note: GC assumes turbulent eddy behavior — perhaps problematic with weakly nonlinear eddies

Gaussian eddy closure

In Fourier space, eddy equations can be written

$$\mathrm{d}\begin{pmatrix}\hat{\psi}_1\\ \hat{\psi}_2\end{pmatrix} = \mathbf{L}_{\boldsymbol{k}}\begin{pmatrix}\hat{\psi}_1\\ \hat{\psi}_2\end{pmatrix}\mathrm{d}\tau + \sigma_{\boldsymbol{k}}\,\mathrm{d}\boldsymbol{W}_{\boldsymbol{k}}$$

Wiener process

$$\mathbf{L}_{\boldsymbol{k}} = -(\gamma_k + \nu k^8)\mathbf{I} + \mathbf{Q}_k^{-1}\left(-i\begin{bmatrix}\overline{\boldsymbol{U}}_1\cdot\boldsymbol{k} & 0\\ 0 & \overline{\boldsymbol{U}}_2\cdot\boldsymbol{k}\end{bmatrix}\mathbf{Q}_k + \begin{bmatrix}-i\boldsymbol{k}\times\nabla\overline{Q}_1 & 0\\ 0 & rk^2 - i\boldsymbol{k}\times\nabla\overline{Q}_2\end{bmatrix}\right)$$

$$\mathbf{Q}_k = \begin{bmatrix}-(\frac{k_d^2}{2}+k^2) & \frac{k_d^2}{2}\\ \frac{k_d^2}{2} & -(\frac{k_d^2}{2}+k^2)\end{bmatrix}.$$

But eddy fluxes needed for resolved model depend only on products.
Use Ito's formula to construct evolution equation for covariances

$$\frac{\mathrm{d}}{\mathrm{d}\tau}C_{\boldsymbol{k}} = \mathbf{L}_{\boldsymbol{k}}C_k + C_k\mathbf{L}_{\boldsymbol{k}}^* + \sigma_{\boldsymbol{k}}\sigma_{\boldsymbol{k}}^* \qquad C_{\boldsymbol{k}} = \mathbb{E}\begin{bmatrix}|\hat{\psi}_1|^2 & \hat{\psi}_1\hat{\psi}_2^*\\ \hat{\psi}_1^*\hat{\psi}_2 & |\hat{\psi}_2|^2\end{bmatrix}$$

Gaussian eddy closure

In Fourier space, eddy equations can be written

$$\mathrm{d}\begin{pmatrix}\hat\psi_1\\ \hat\psi_2\end{pmatrix} = \mathbf{L}_{\boldsymbol{k}}\begin{pmatrix}\hat\psi_1\\ \hat\psi_2\end{pmatrix}\mathrm{d}\tau + \boxed{\sigma_{\boldsymbol{k}}}\,\mathrm{d}\boldsymbol{W}_{\boldsymbol{k}}$$

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$$\mathbf{Q}_k = \begin{bmatrix}-\left(\frac{k_d^2}{2}+k^2\right) & \frac{k_d^2}{2}\\ \frac{k_d^2}{2} & -\left(\frac{k_d^2}{2}+k^2\right)\end{bmatrix}.$$

But eddy fluxes needed for resolved model depend only on products.
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Need closure for these terms

Gaussian eddy closure

The equilibrium solution for the covariance, in the absence of mean gradients, is used as the initial condition. The following properties are demanded:

- isotropic energy spectrum $\Rightarrow$ zero mean Reynolds stress terms
- energy spectrum $\propto k^{-5/3}$ for $k < k_d$
- energy spectrum $\propto k^{-3}$ for $k > k_d$
- eddies do not generate heat flux in the absence of temperature gradient
- a constant ratio of barotropic and baroclinic energy at each k

The damping parameter is set equal to the nonlinear inverse eddy timescale at each k,

$$\gamma_k = \gamma_0 \sqrt{k^3 E(k)}$$

The total energy in the initial condition is a tunable parameter... Various methods are examined in the papers.

Correlated stochastic plane waves

$$\overline{u_1'(\psi_2' - \psi_1')} = \int_0^{2\pi} \int_{k_0}^{k_{\max}} k^2 \sin(\theta) \left(\epsilon \int_0^{\epsilon^{-1}} \mathbb{E}[\mathcal{I}\{\hat{\psi}_1 \hat{\psi}_2^*\}] \, \mathrm{d}\tau \right) \mathrm{d}k \, \mathrm{d}\theta,$$

$$\overline{u_i' v_i'} = \frac{1}{2} \int_0^{2\pi} \int_{k_0}^{k_{\max}} k^3 \sin(2\theta) \left(\epsilon \int_0^{\epsilon^{-1}} \mathbb{E}[|\hat{\psi}_i|^2] \, \mathrm{d}\tau \right) \mathrm{d}k \, \mathrm{d}\theta.$$

- If eddies are isotropic, Reynold's stress terms are 0.
- Heat flux results from anisotropy
- Choose random eddy angle along which to integrate eddy stress terms — resulting fluxes are therefore not always downgradient

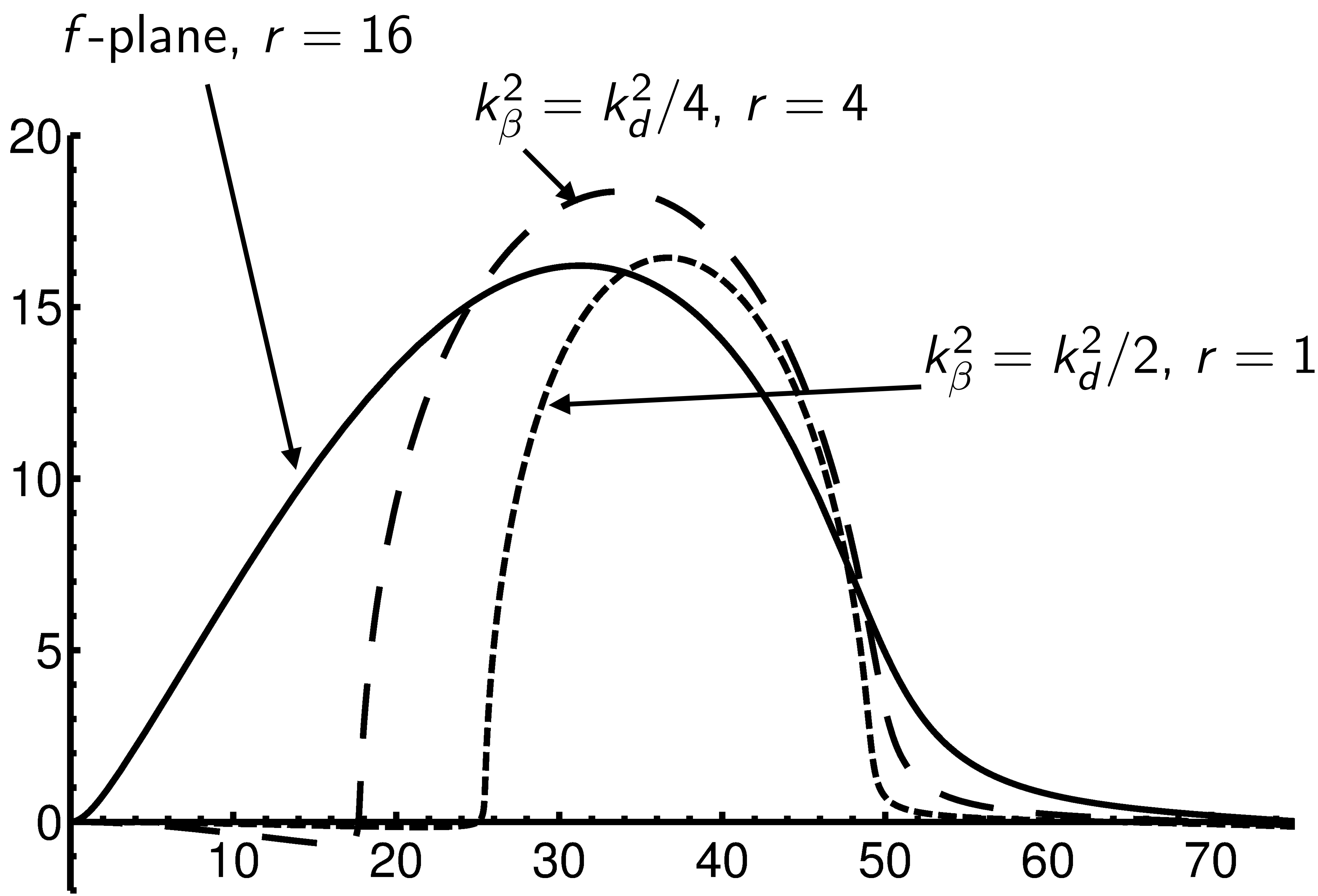
The Algorithm

1. At the beginning of a coarse model time step, evaluate the large-scale variables that appear in the eddy equations, e.g. shear, vorticity gradient.
2. Pick a random direction for the eddies.
3. While holding the large-scale terms fixed, evolve the eddies for a fixed time of length ϵ^{-1}. This is cheap because eddy dynamics are linear.
4. Compute the eddy PV flux from step 3. This is also cheap because of simple Fourier analysis.
5. Update the large-scale variables.
6. Re-set the eddy variables to a 'climatological' state, i.e. forget the final state of the eddies from the end of step 3.
7. Repeat

Test in doubly-periodic case

- Run eddy-resolving reference simulations with $k_d = 50$ and a 512^2 grid.
- Then we run stochastic SP using the same code on a 64^2 grid — 8X lower resolution.
- The coarse-grid Nyquist wavenumber is 32, which is smaller than the deformation radius but larger than the peak of the KE spectrum (which is around k=5).
- Since our coarse grid is larger than the deformation scale, we need to simultaneously parameterize the downscale cascade of APE and the inverse cascade of KE.

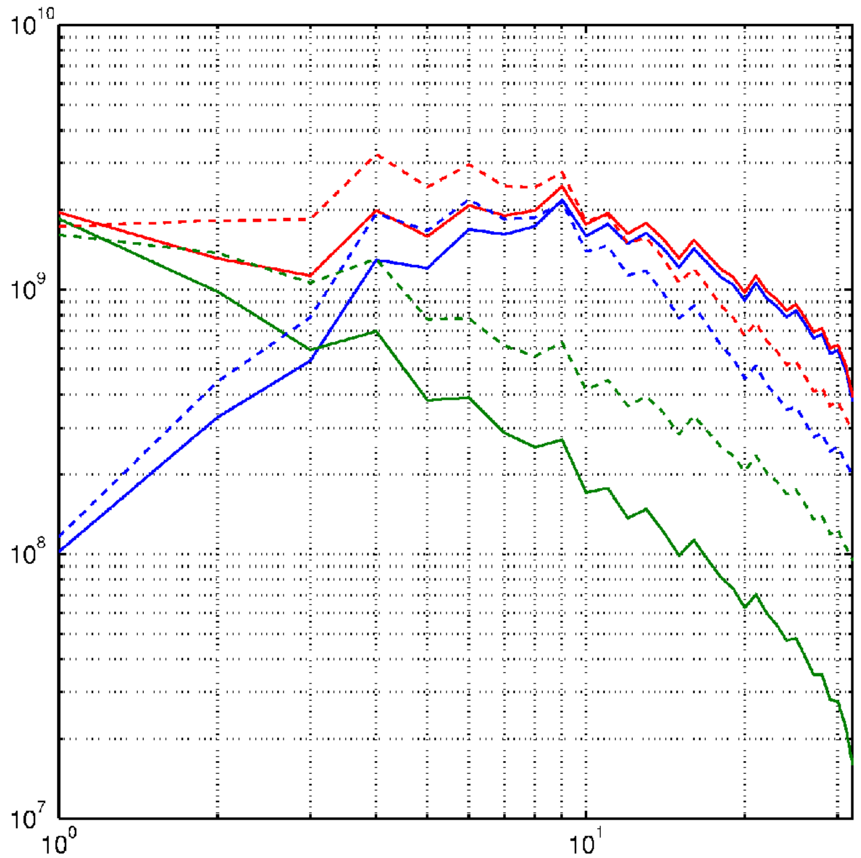
Test in doubly-periodic case



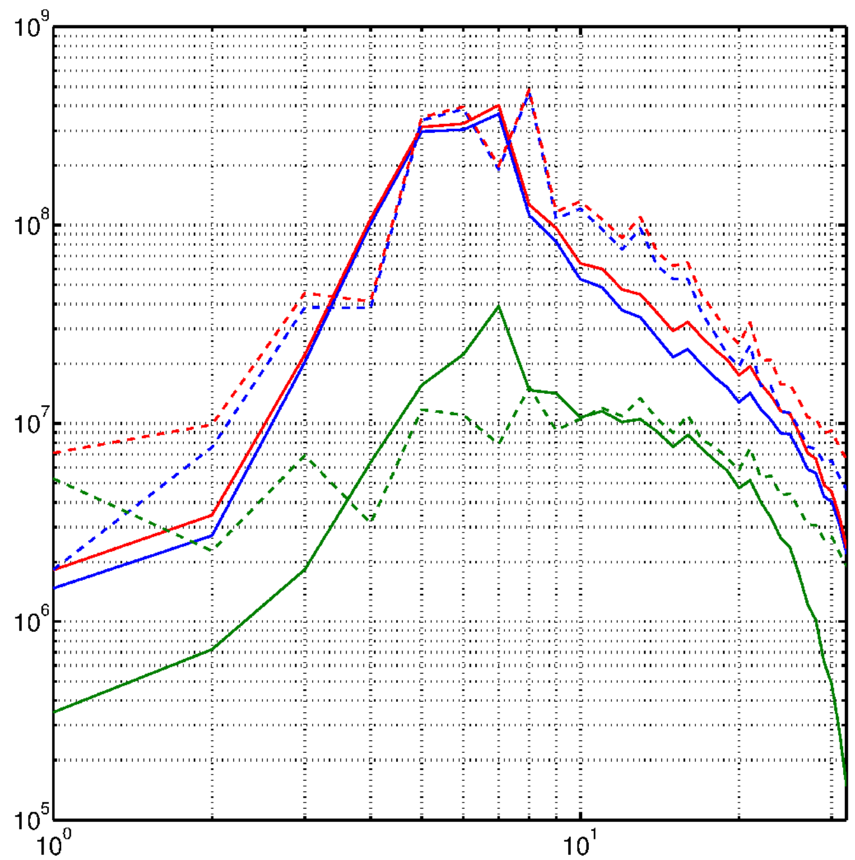
The growth rates of linear instability versus k_x ($k_y = 0$).

Test in doubly-periodic case

f-plane, $r = 16$



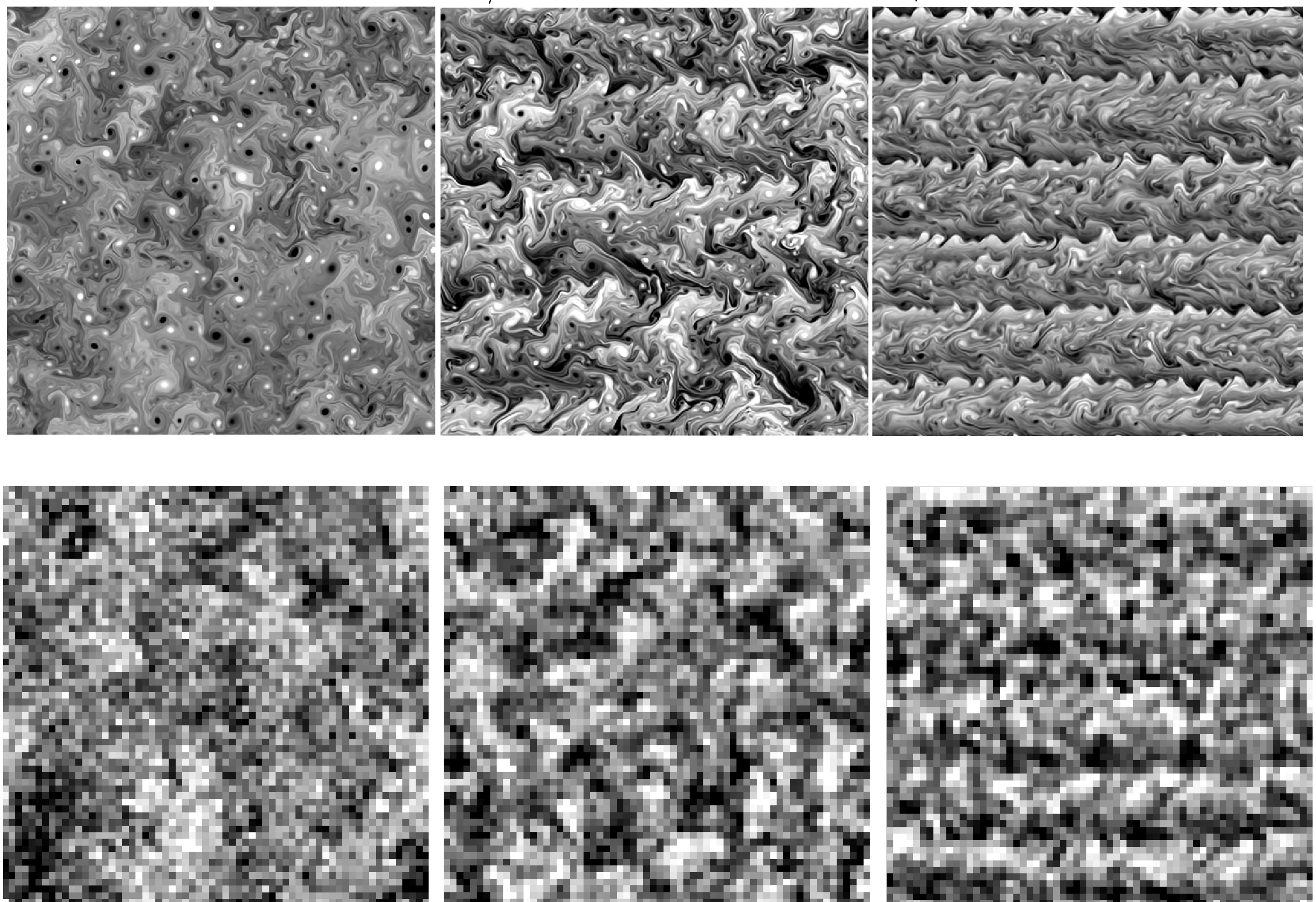
$k_\beta^2 = k_d^2/2$, $r = 1$



Time-averaged 1D energy spectra for SP (solid) and DNS (dashed). KE (blue), APE (green), total (red)

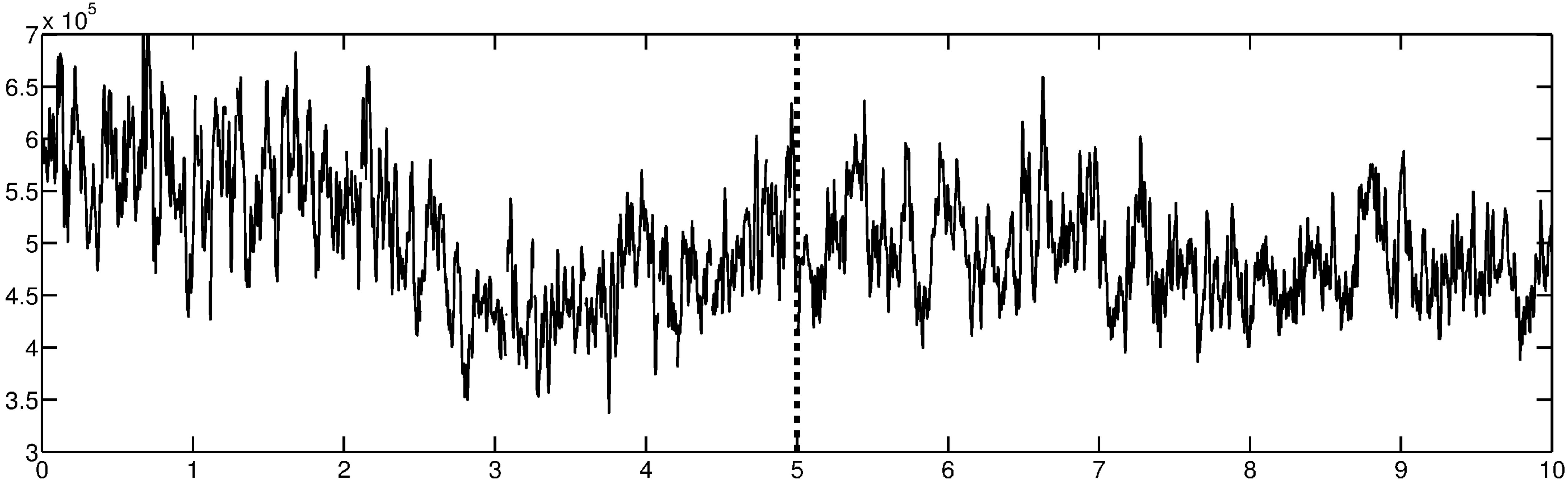
Test in doubly-periodic case

f-plane, $r = 16$ $\quad$ $k_\beta^2 = k_d^2/4$, $r = 4$ $\quad$ $k_\beta^2 = k_d^2/2$, $r = 1$



Heat flux in f-plane case

DNS SP



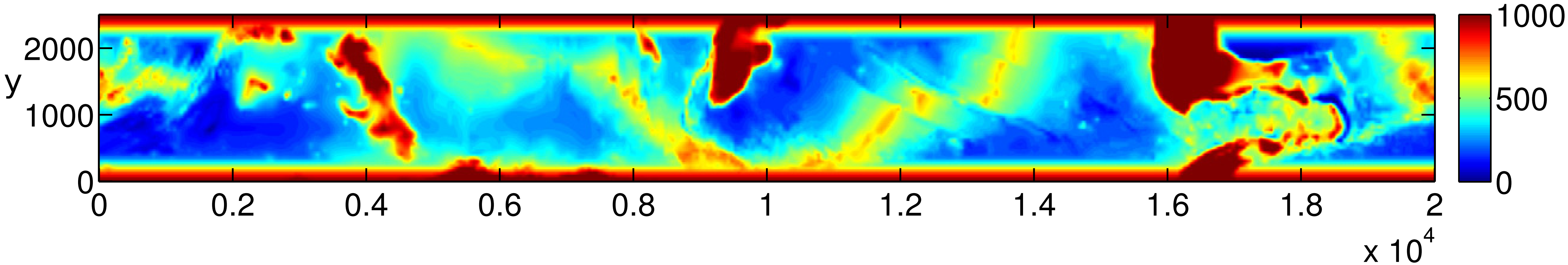
SP model reproduces heat flux of DNS case, despite complete absence of small-scale vortices present in DNS

Test in channel with ACC topography

- Compare SP to four variants of Gent-McWilliams in a 2-layer QG channel model with ACC topography

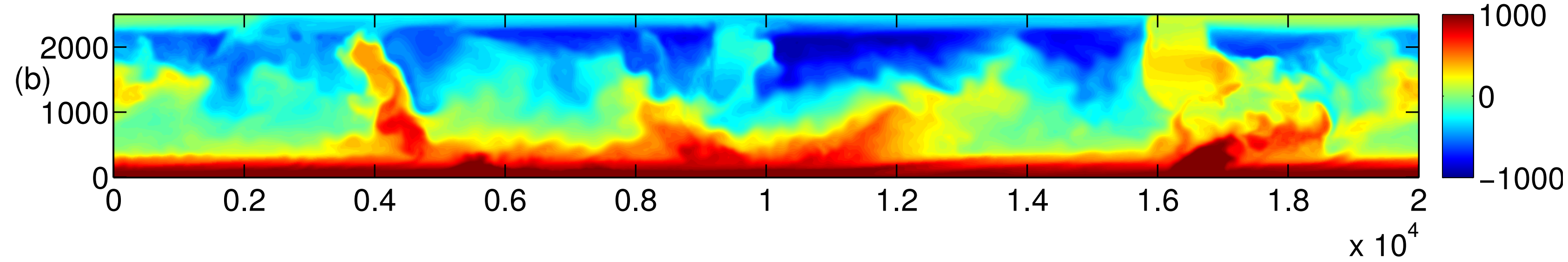
- Domain size is 20,000 X 2,500 km X 2 km layer depths. R_d = 12.25 km, and dx = 6.5 km for the eddy-resolving reference solution.

- Coarse model uses same code but has grid 8 times coarser (dx = 50 km): barely eddy permitting.

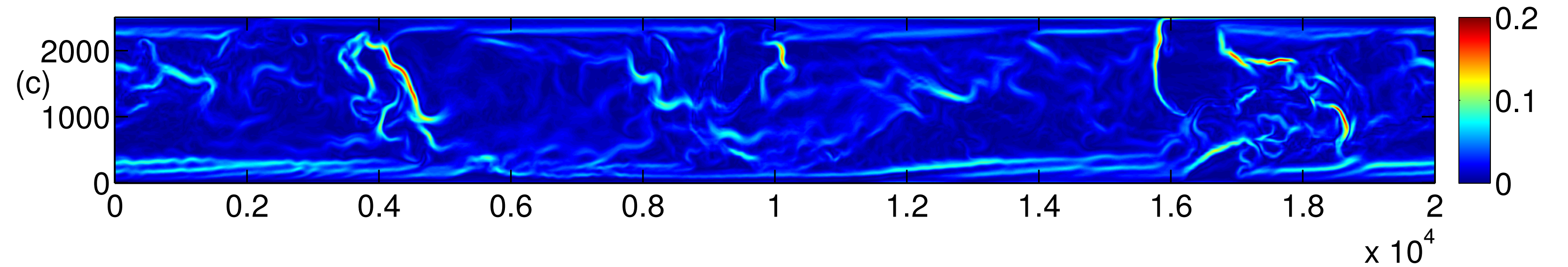


Reference simulation

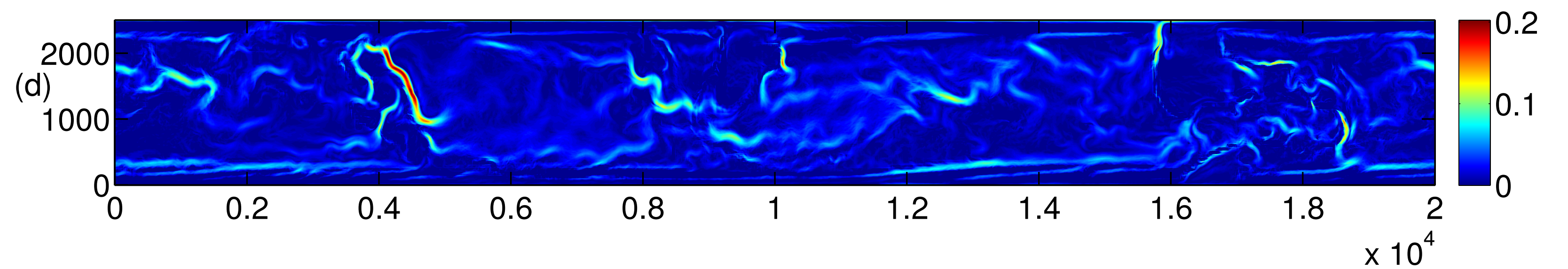
<Interface height> (m)



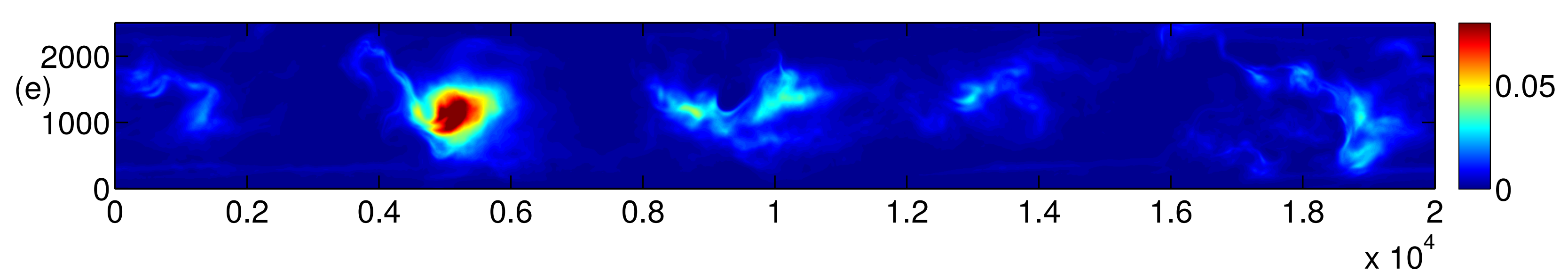
<Shear> (m/s)



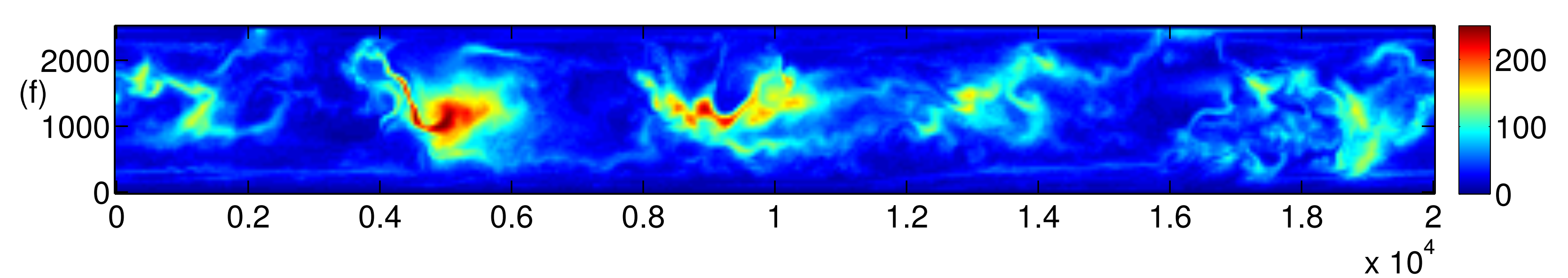
growth rate (days^{-1})



EKE (m/s)^2



STD int height (m)



QG with GM

$$\partial_t q_2 + J[\psi_2, q_2 + \beta y] = \frac{2f_0^2}{g'H}\nabla\cdot(\kappa\nabla(\psi_1-\psi_2)) - \frac{f_0}{H_2}J[\psi_2, h_b] - r\nabla^2\psi_2 + \nu_2\nabla^4\psi_2$$

and likewise for layer 1

Flavors of GM

0. κ is a tunable constant (Gent McWilliams 90)

1. $\kappa = \alpha L_d \Delta U$ (Stone 72)

2. $\kappa = \alpha \frac{(\Delta U)^2}{r}$ (Cessi 08)

3. $\kappa = \alpha \frac{(\Delta U)^3}{\beta^2 L_d^3}$ $\quad \beta_{\text{eff}} = \left|\beta\hat{\mathbf{y}} + \frac{f_0}{H}\nabla h_b\right|$ (Held Larichev 96)

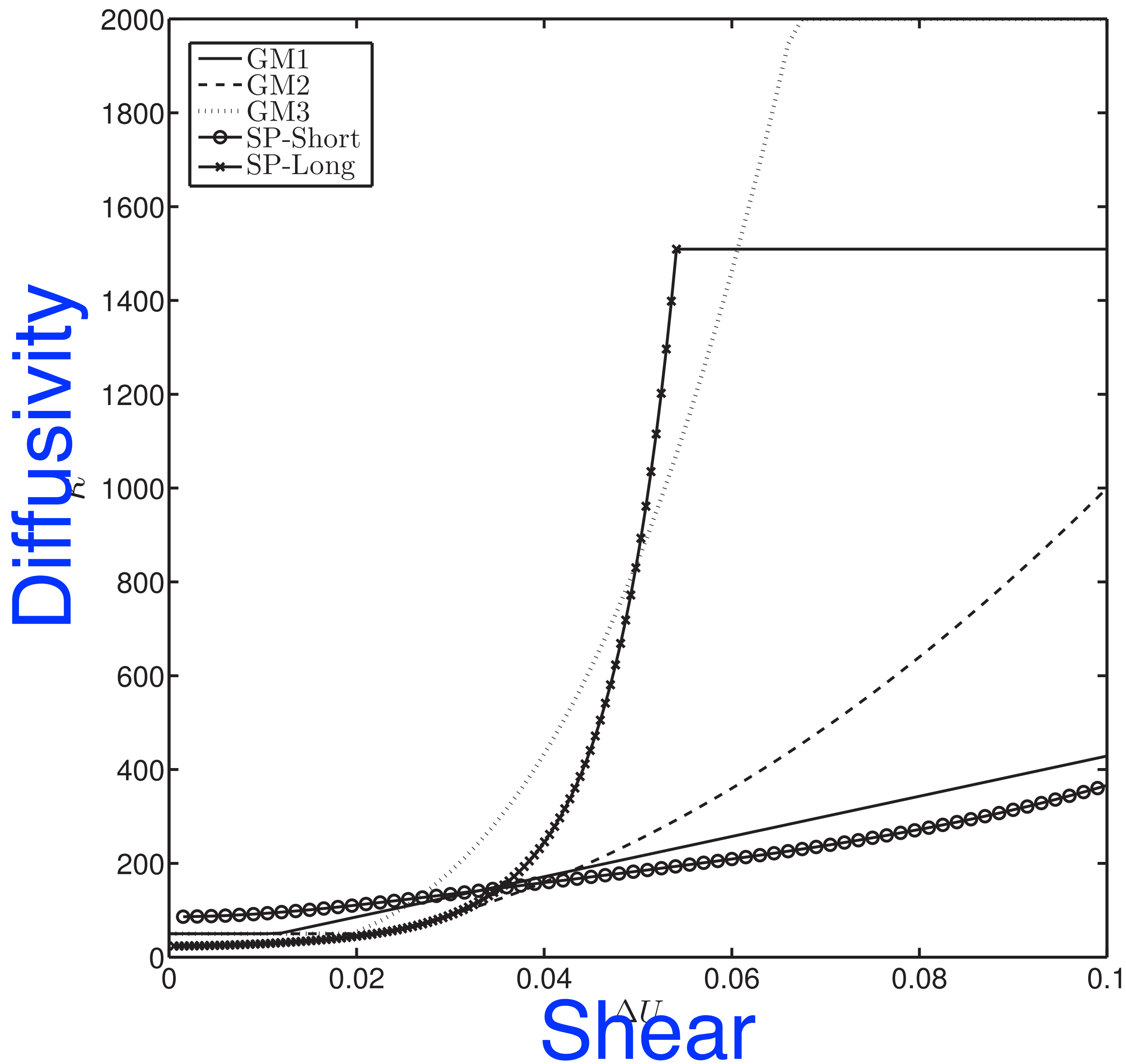
4. $\kappa = \alpha \frac{L_z^2}{T}$ $\quad \frac{1}{T} = \frac{|f_0|}{\sqrt{Ri}} = \Delta U\sqrt{\frac{2f_0^2}{g'H}}.$ (Visbeck et al 97)

Effective SP diffusivity

One can compute an effective diffusivity for SP, by restricting to cases where eddy angle results in down-gradient fluxes:

$$\kappa_{\text{eff}} = -\frac{\overline{v'_1(\psi'_2 - \psi'_1)}}{2U}$$

One can then compare all flavors of GM with SP:



Metric for comparison

How do we measure performance of the coarse models?

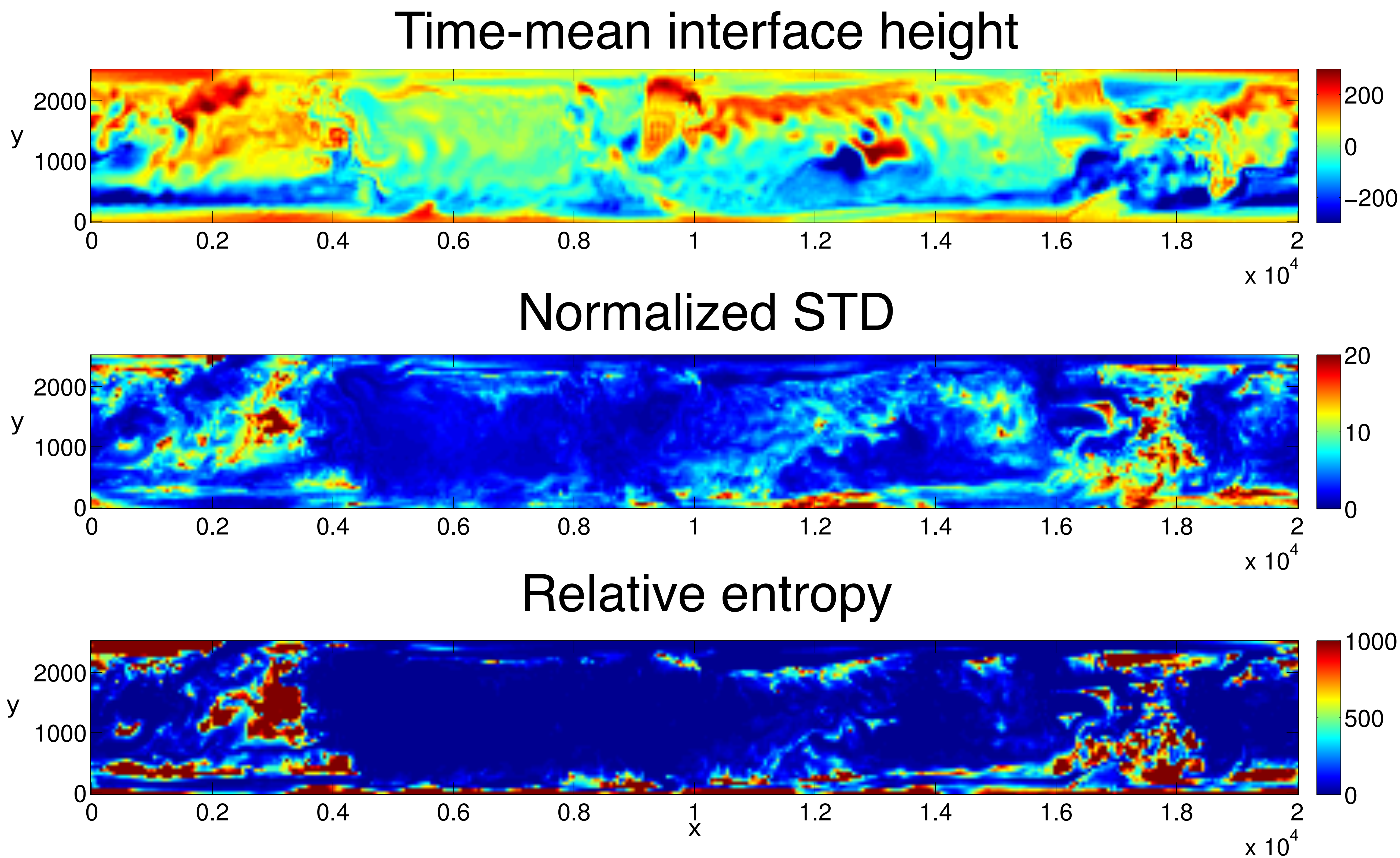
- X The barotropic transport in all models is comparable and too high by $\approx 25\%$; due to under-resolution of topographic form stress.
- X All models are tuned to have correct area- and time-averaged zonal baroclinic shear.
- We look at RMS error in the time-mean layer interface height (pattern correlations are extremely high for all models)
- We look at the local temporal variability of the layer interface height
  - Area-averaged temporal standard deviation of the interface height σ
  - RMS error in σ
  - Pattern correlation in σ
  - Relative entropy for climatological distribution of interface height

Flavors of SP

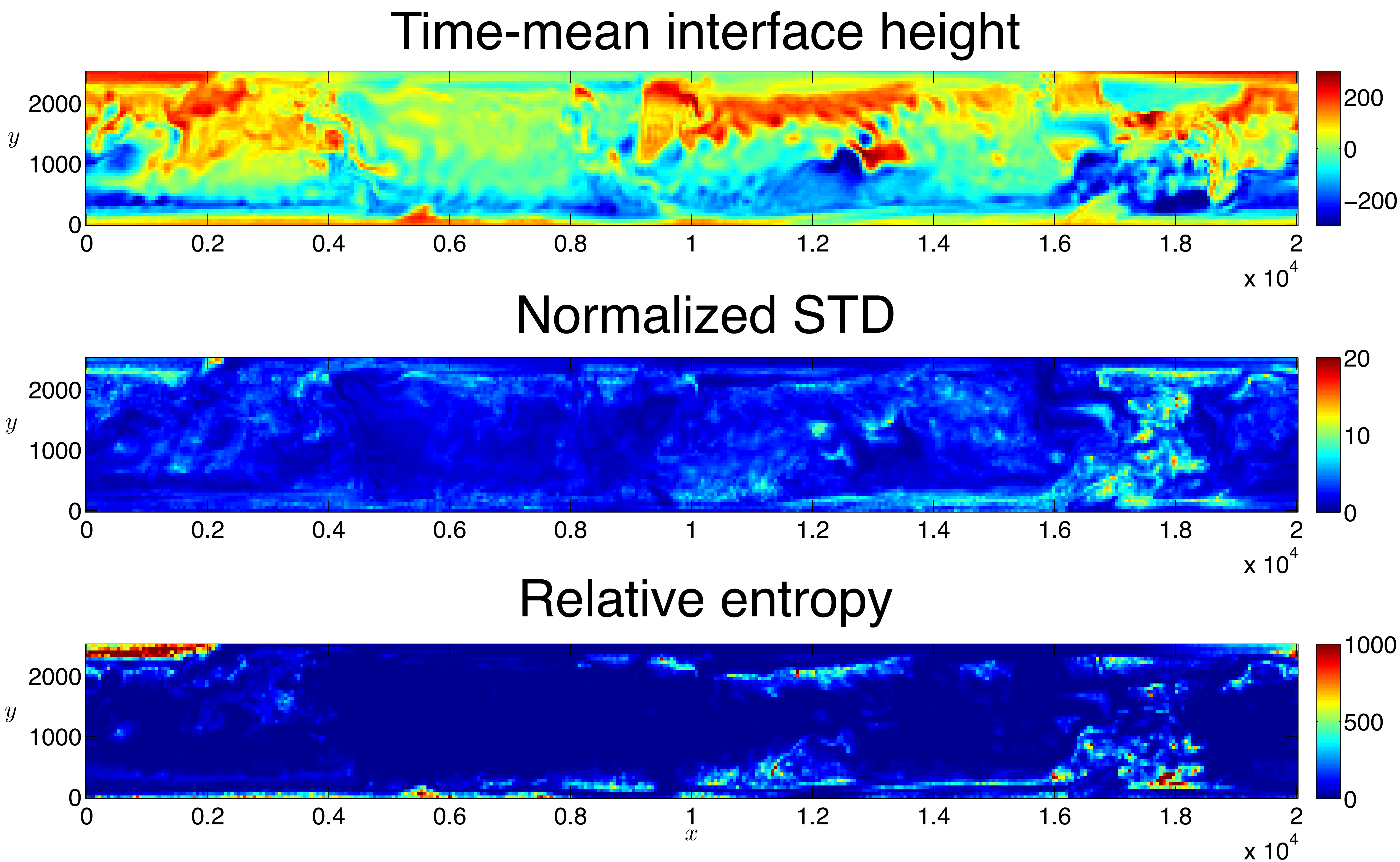
We tested 8 configurations of stochastic SP

- Short and long eddy evolution times, which also have different γ_0.
- Reynolds stresses included or ignored in large-scale model
- Large-scale vorticity gradient, β, and topographic gradient included or ignored in eddy equations

GM4 (best) results



SP ‘short’ (best) results



Summary of results

- All the models have too little variability
- Models with more-accurate time-mean typically have less-accurate variability
- Of the GM models GM3 had best mean and worst variability; GM0 had worst mean and best variability; GM4 had best relative entropy
- SP models with Reynolds stresses had much better (higher) variability
- SP models with long average had better mean but worse variability compared to short average
- Best SP models have comparable mean to best GM model, but better variability which leads to best overall relative entropy/climate fidelity.

Future directions

- In a nutshell, SP is GM + backscatter
- SP can be improved by including TKE transport equation to set local energies
- Implementation in PE model with high vertical resolution could proceed with projection of eddy model onto a few vertical modes

Domain-averaged measures of skill for the GM parameterizations. The RMS of σ and the RMS errors are measured in meters, the remaining columns are dimensionless. The RMS of σ for the reference simulation is 58 m.

	RMSE of $\overline{\eta}$	RMS of σ	RMSE of σ	PC of σ^2	Dispersion	Rel. Ent.
GM0	131	44	33	0.80	28	995
GM1	105	33	37	0.77	63	1080
GM2	95	31	37	0.77	51	719
GM3	78	26	40	0.69	95	617
GM4	112	36	35	0.77	18	362

Domain-averaged measures of skill for stochastic SP. The RMS of σ and the RMS errors are measured in meters, the remaining columns are dimensionless. The RMS of σ for the reference simulation is 58 m.

		RMSE of $\overline{\eta}$	RMS of σ	RMSE of σ	PC of σ^2	Dispersion	Rel. Ent.
SP-short	NRS-f	116	33	37	0.78	64	1417
	NRS-β	126	39	33	0.81	25	731
	RS-f	104	39	31	0.80	4	62
	RS-β	115	42	29	0.81	4	75
SP-long	NRS-f	87	28	39	0.76	39	455
	NRS-β	87	30	37	0.78	27	382
	RS-f	82	32	36	0.74	19	225
	RS-β	82	33	35	0.77	22	229