Energetically consistent ocean models

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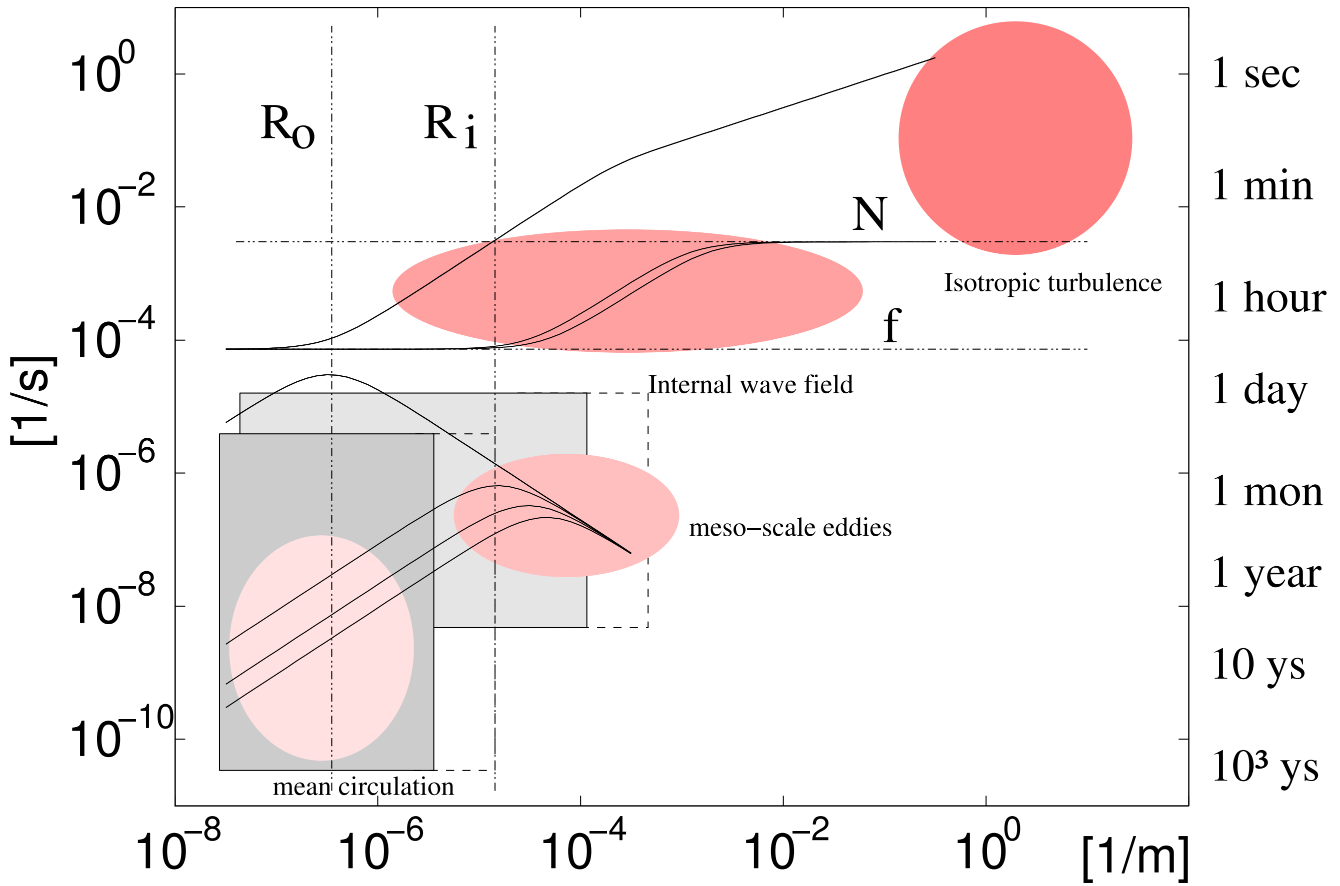
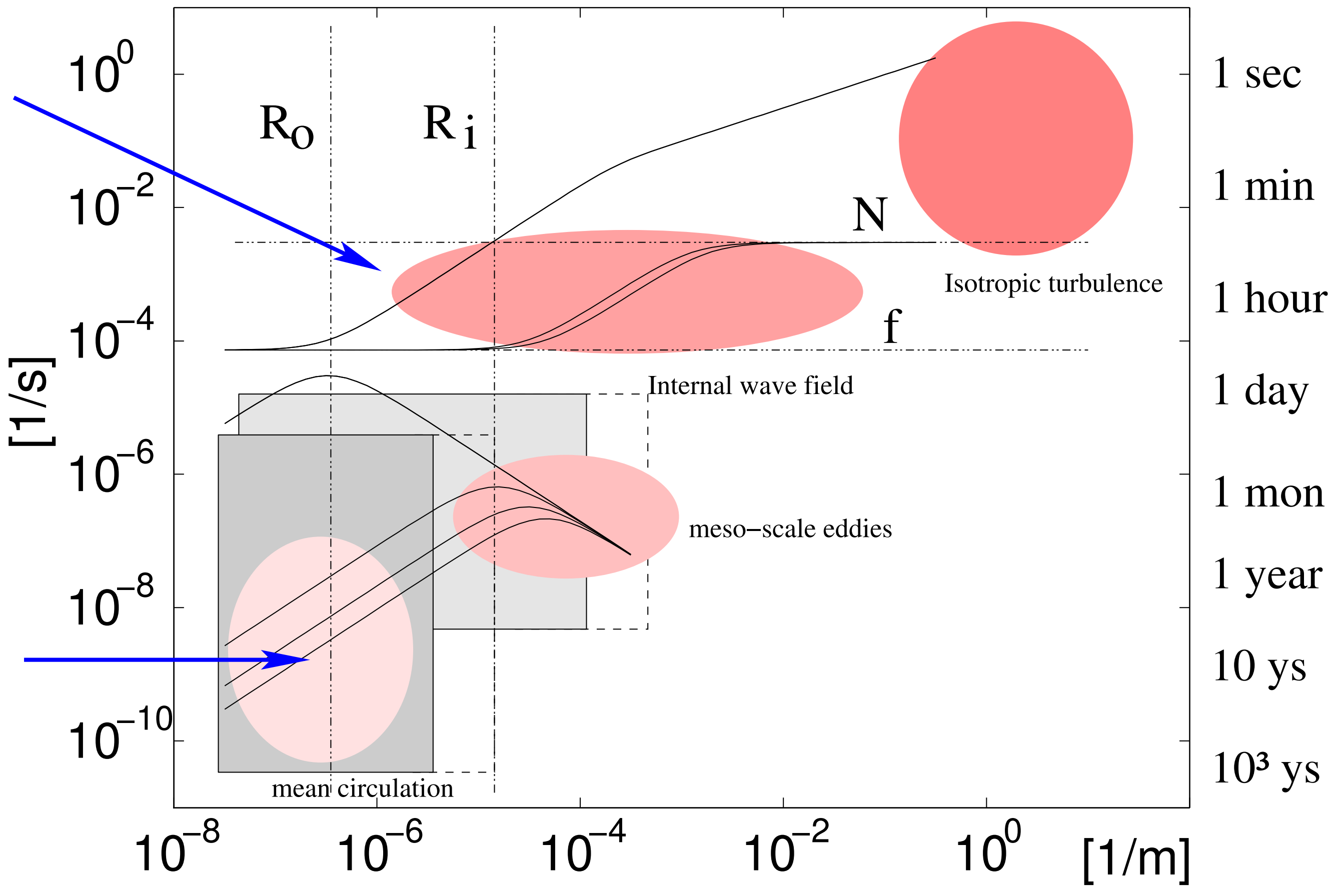
dynamical regimes



- solid lines: dispersion relations of linear wave solutions
- red ellipses: dynamical regimes, grey boxes: ocean models
- R_o, R_i: Rossby radii of deformation, N stability frequency, f Coriolis frequency

dynamical regimes



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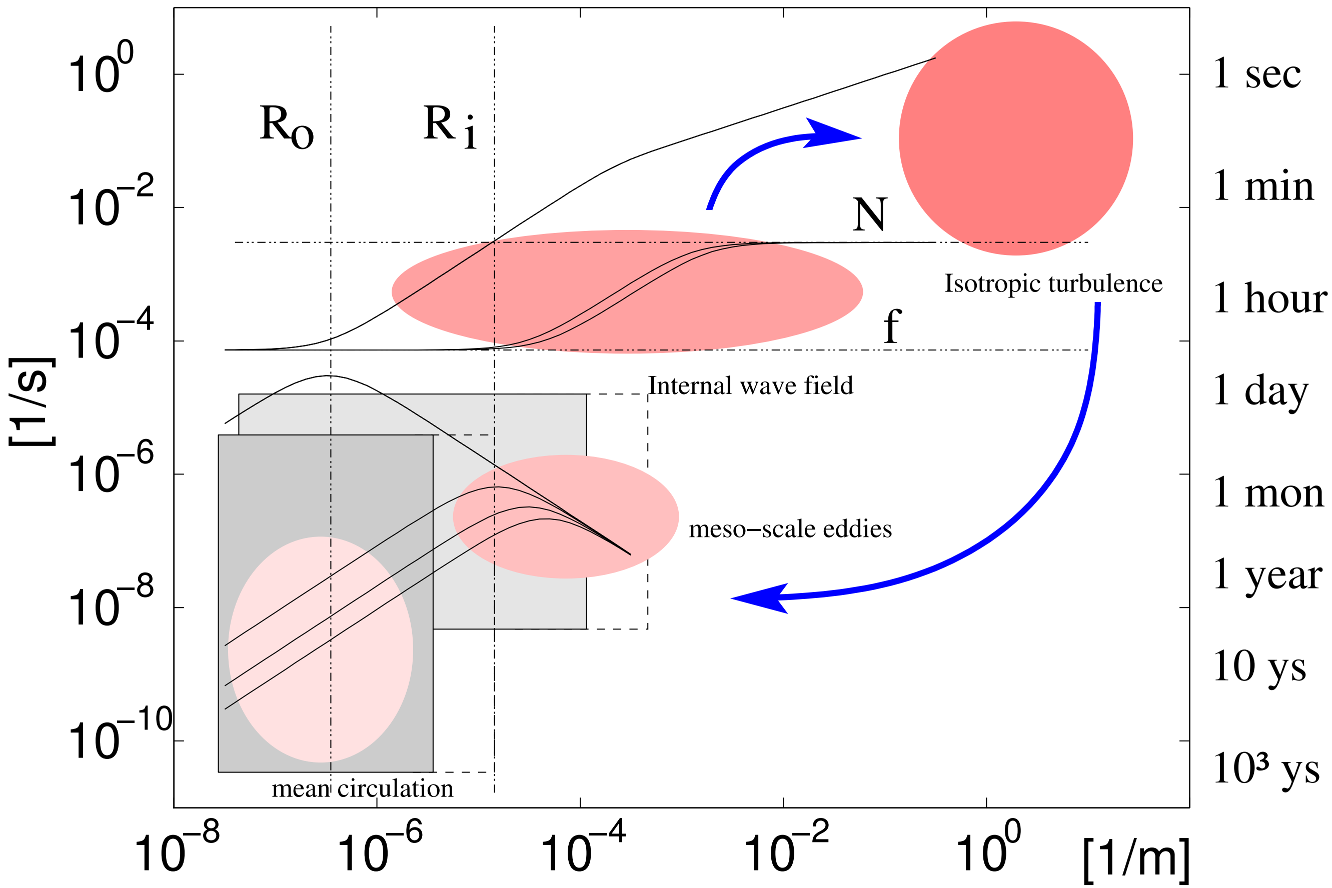

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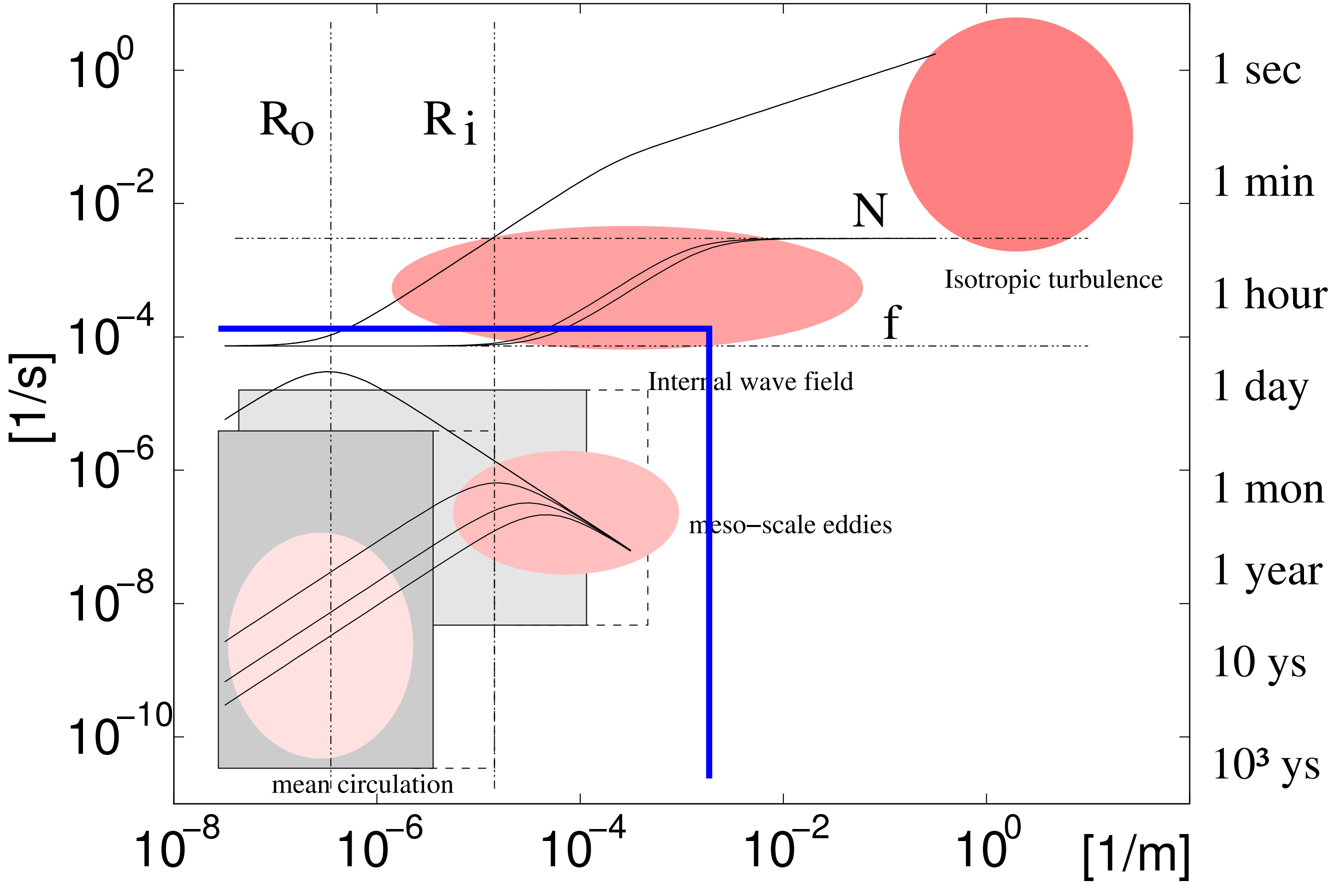

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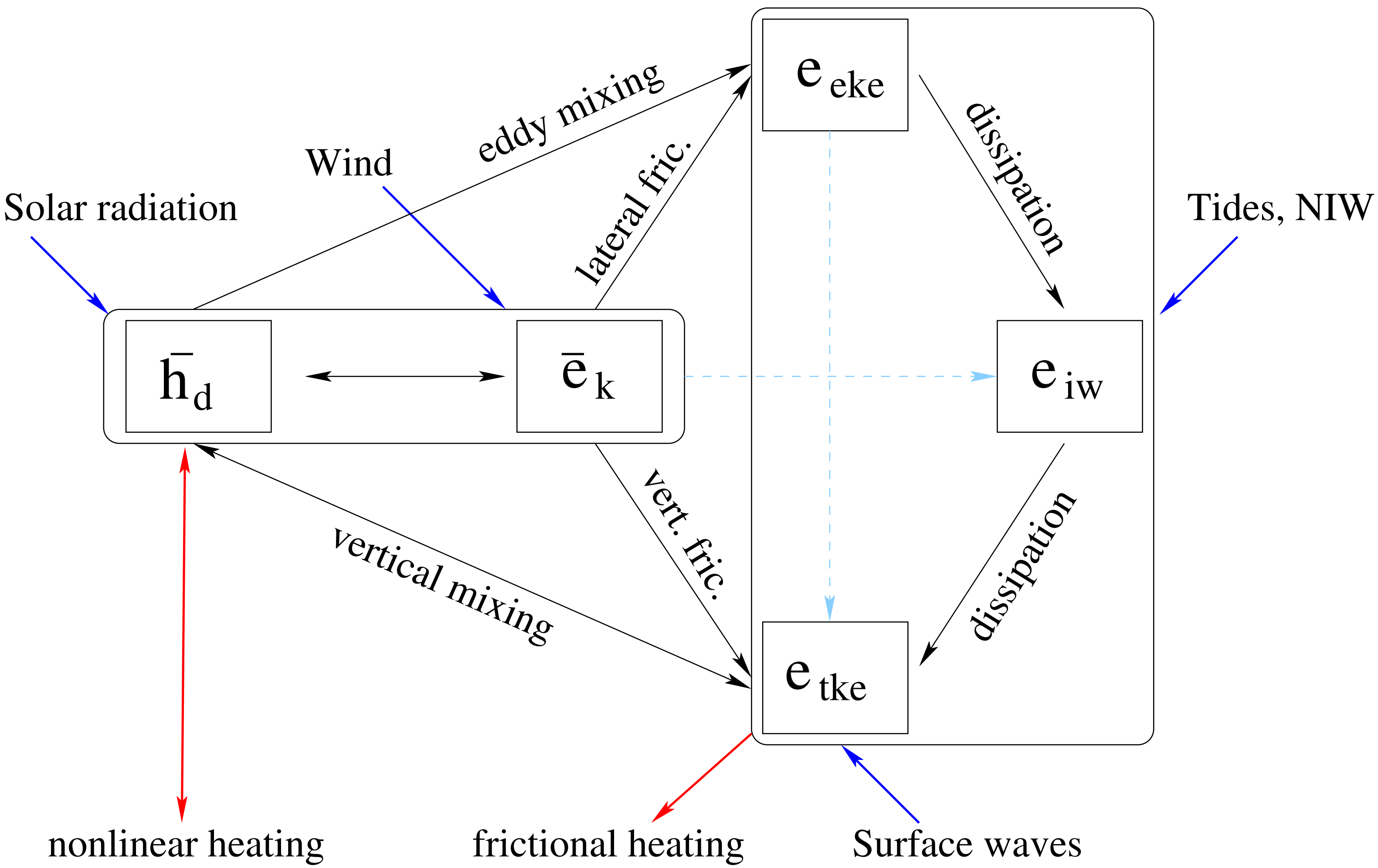
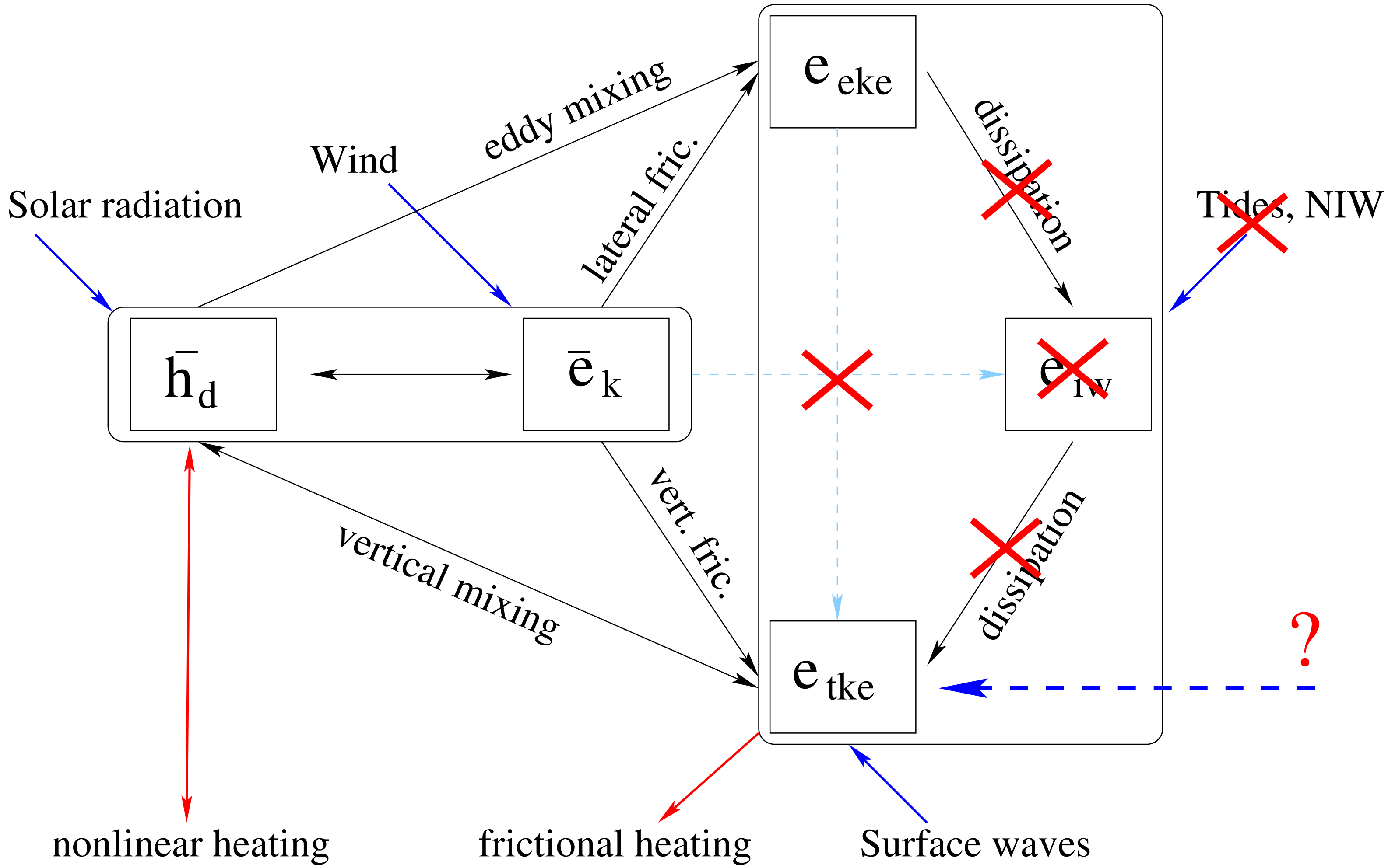
consistent energy cycle of an ocean model



- forcing by blue, dissipation by red arrows

inconsistent energy cycle of an ocean model



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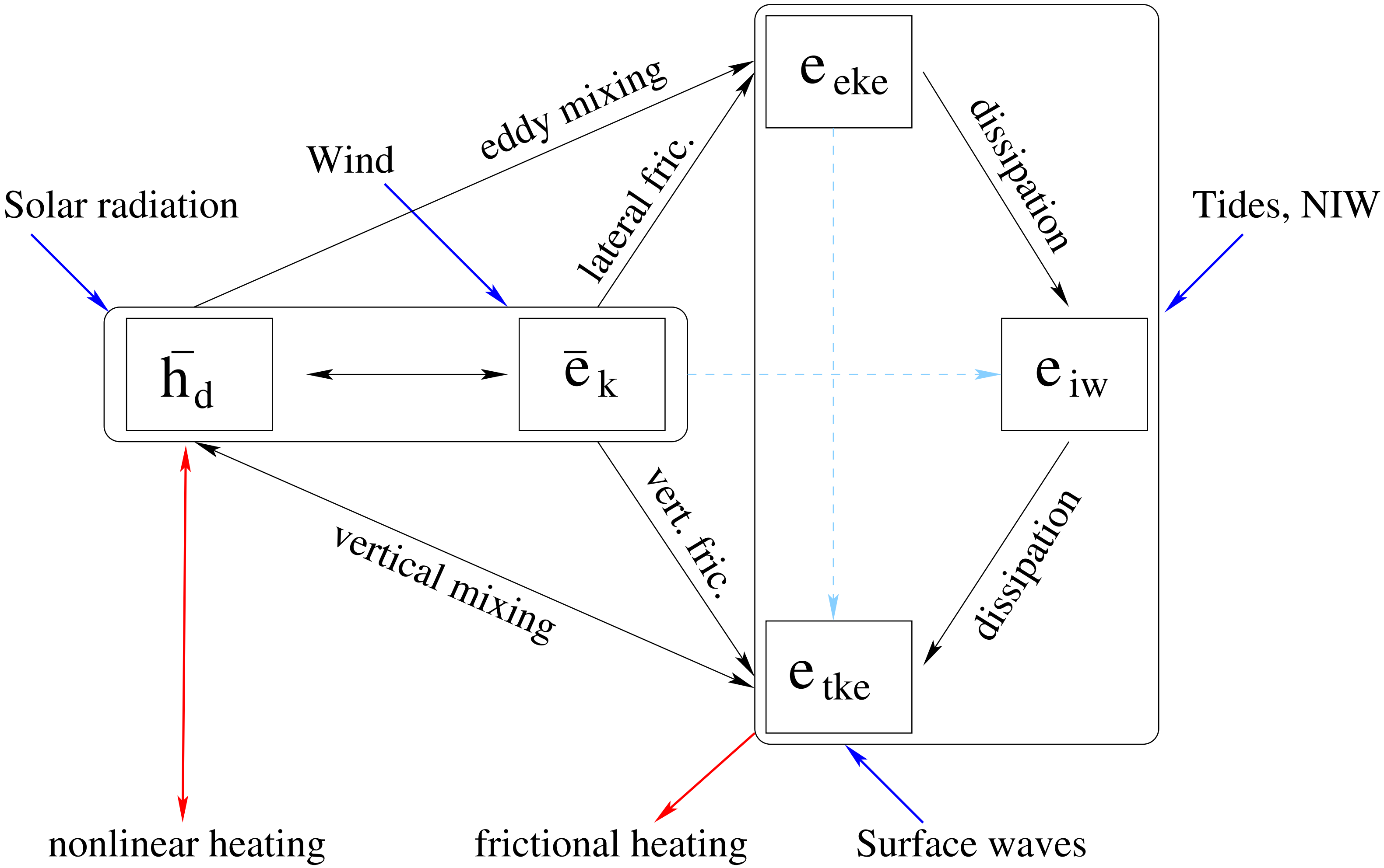


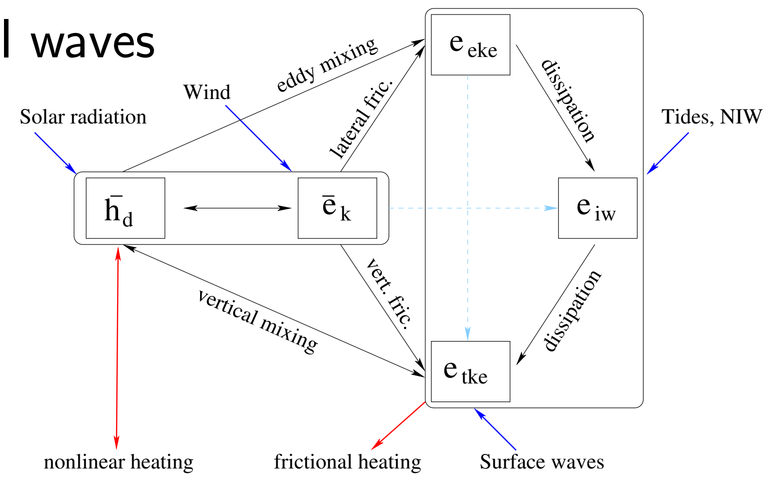
- forcing by blue, dissipation by red arrows

TKE model by Gaspar et al (1990)

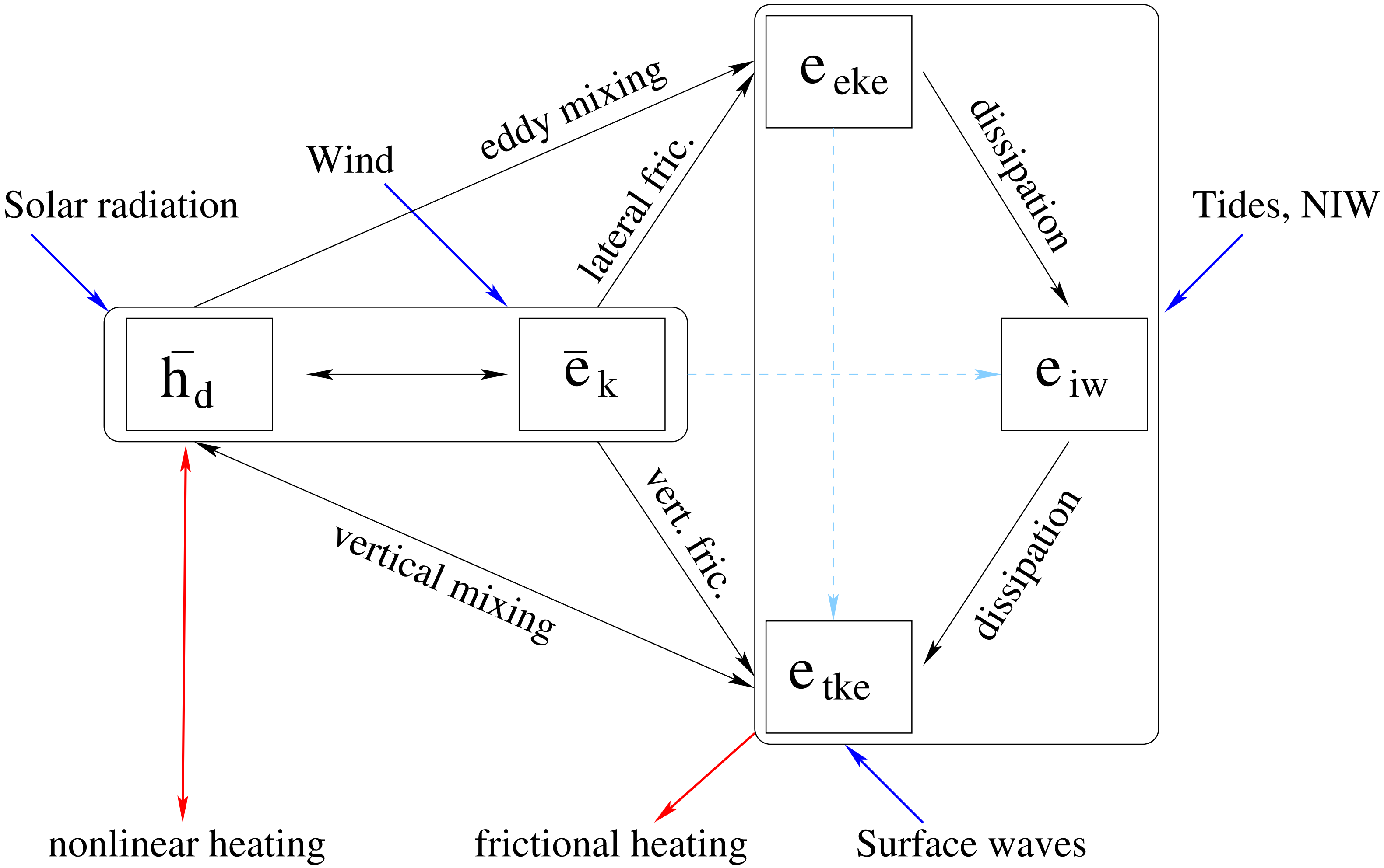
- mixed layer closure model based on e_{tke}

$$\frac{de_{tke}}{dt} = -\boldsymbol{\nabla}\cdot(..) + c_u K \left(\frac{\partial \bar{\boldsymbol{u}}}{\partial z}\right)^2 - c_b K N^2 - c_\epsilon \bar{E}^{3/2} L^{-1}$$

- production by vertical shear $c_u K \left(\partial \bar{\boldsymbol{u}} \partial z\right)^2 \rightarrow \bar{e}_K$
- destruction by vertical mixing $c_b K N^2 \rightarrow \bar{h}_d$
- dissipation by molecular friction $c_\epsilon e_{tke}^{3/2} L^{-1} \rightarrow$ internal energy (heat)
- closure with $K = e_{tke}^{1/2} L$ and $L = \sqrt{2 e_{tke}/N}$
- in the interior production by simulated $\partial \bar{\boldsymbol{u}}/\partial z$ almost vanishes: use minimal threshold e_{min} for e_{tke} and it follows $K = \sqrt{2} e_{min}/N$
- shear production is related to breaking internal waves $\rightarrow$ add dissipation from e_{iw} and set $e_{min} = 0$



consistent energy cycle of an ocean model



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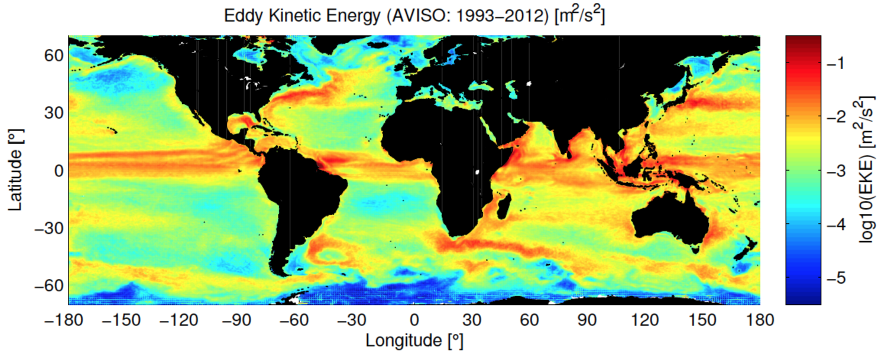
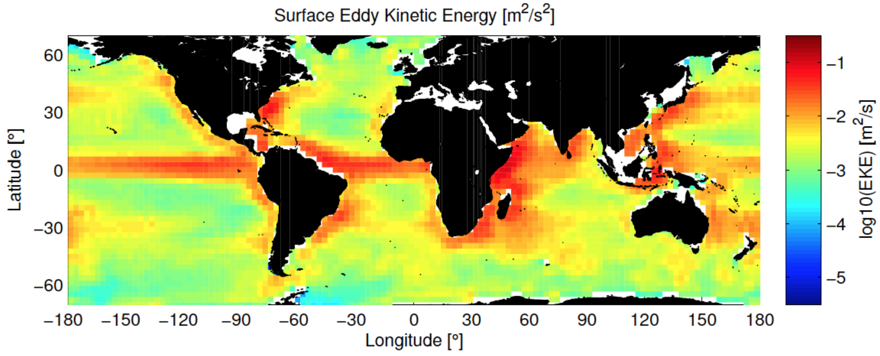
meso-scale eddy closure by Eden and Greatbatch (2008)

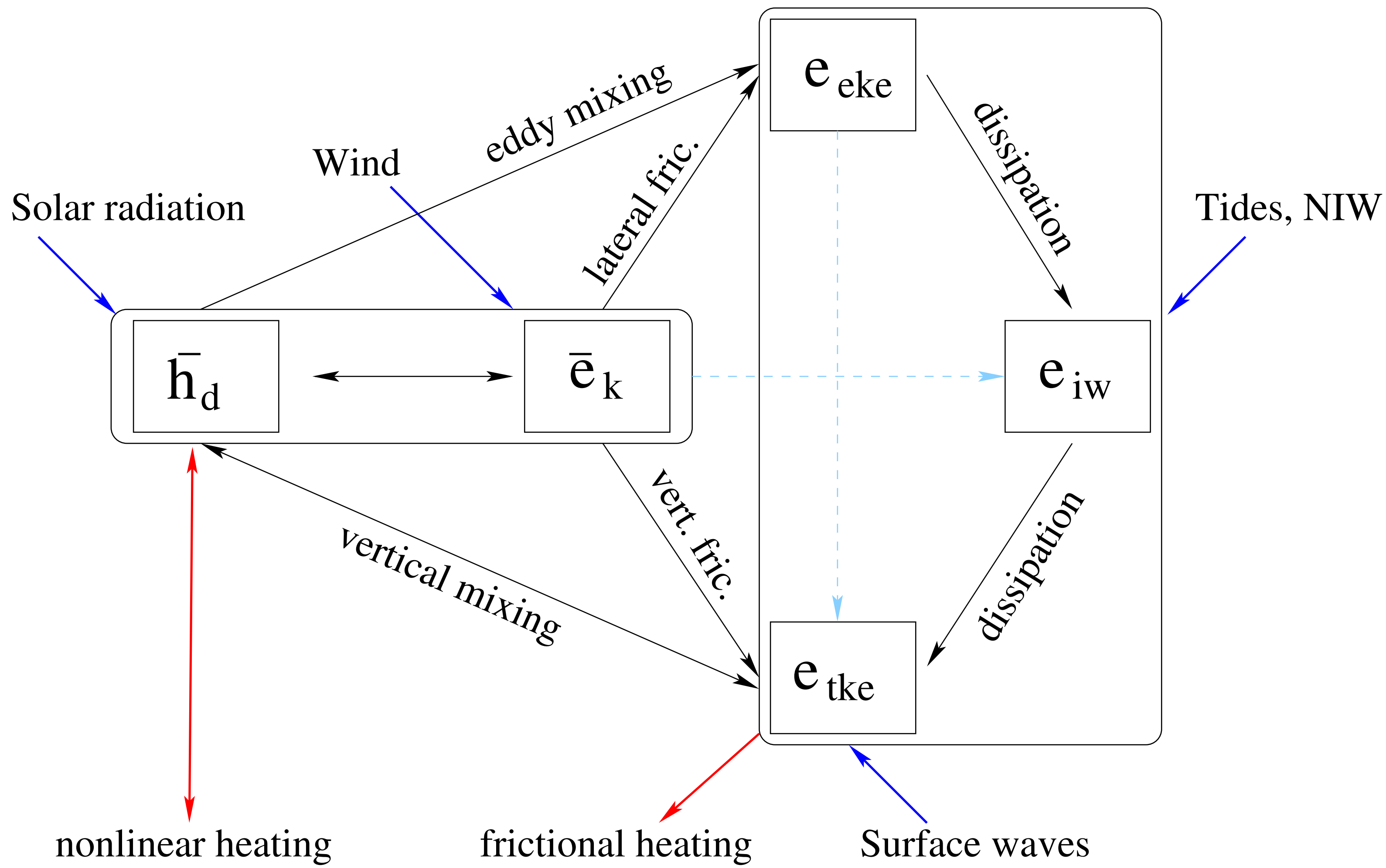
$$\frac{de_{eke}}{dt} = -\nabla \cdot (..) + \text{"lateral shear instab./fric."} + K_{gm}\frac{|\nabla \bar{b}|^2}{N^2} - \epsilon_{eke}$$

- closure $K_{gm} = e_{eke}^{1/2} L$ with $L = \min(L_{Rhines}, L_{Rossby})$
  but better choices for L possible $\rightarrow$ Griesel et al (in prep)
- viscous dissipation by harmonic "lateral friction"
- dissipation of e_{eke} by $\epsilon_{eke} = e_{eke}^{3/2}/L$ or simply $\epsilon_{eke} = r e_{eke}$
  but better closures for ϵ_{eke} are needed (see below)



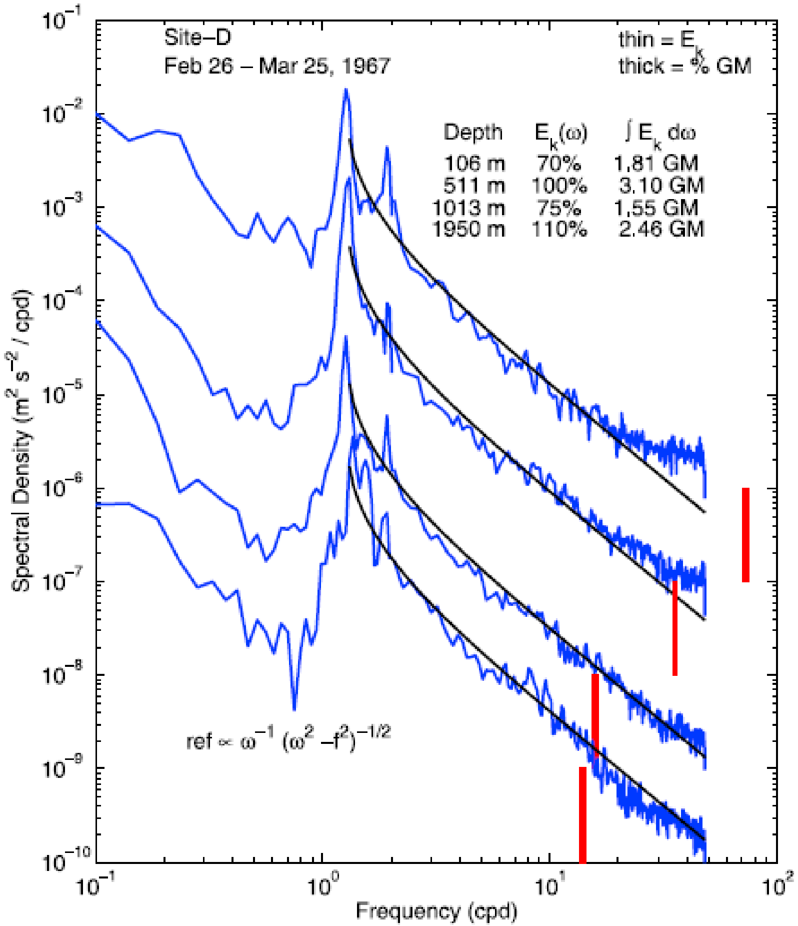


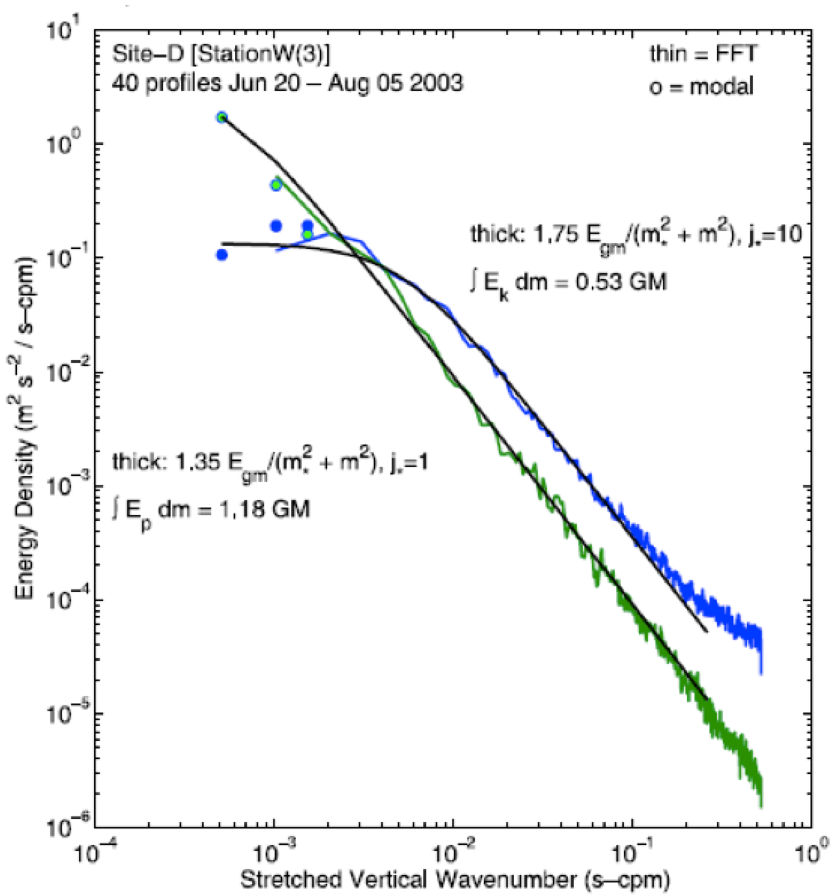
consistent energy cycle of an ocean model



- forcing by blue, dissipation by red arrows

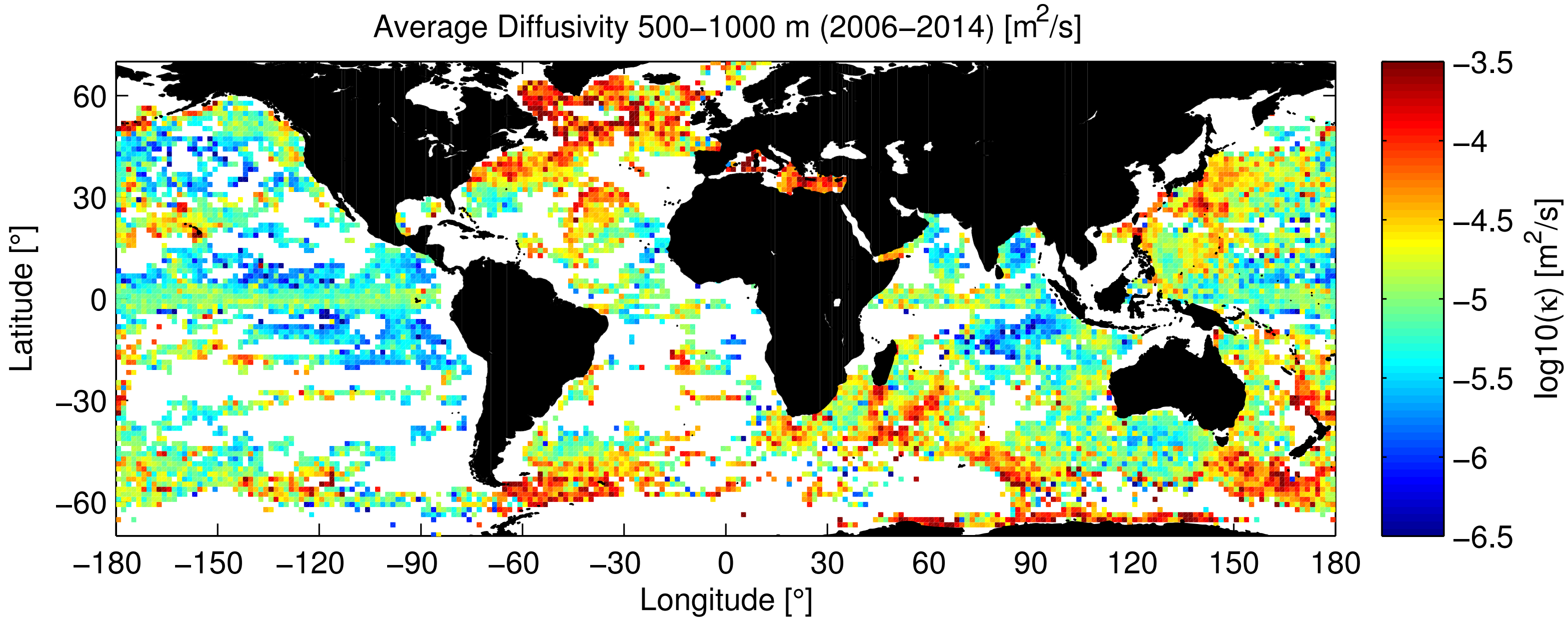
- generic internal wave spectrum in the ocean (GM spectrum)
- Garret and Munk (72, 75): $\mathcal{E}(k_h, \omega, z) \sim \omega^{-2} \sim k_h^{-2} \sim k_3^{-2}$





from Polzin and Lvov (2011)

internal gravity waves and mixing



from Pollmann, Eden and Olbers (2015)

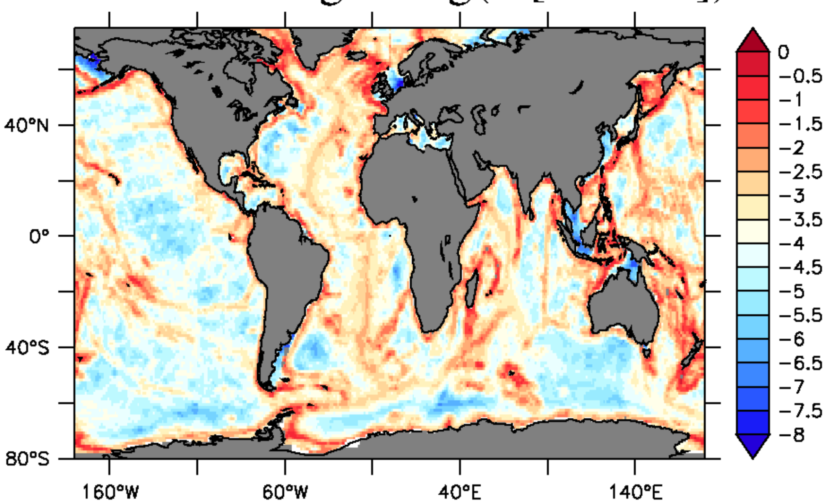
- vertical diffusivity (in $\log_{10} 1/[\mathrm{m}^2/\mathrm{s}]$) estimated from ARGO floats as in Kunze et al (2006), Whalen et al (2012)

IDEMIX by Olbers and Eden (2013), Eden and Olbers (2014)

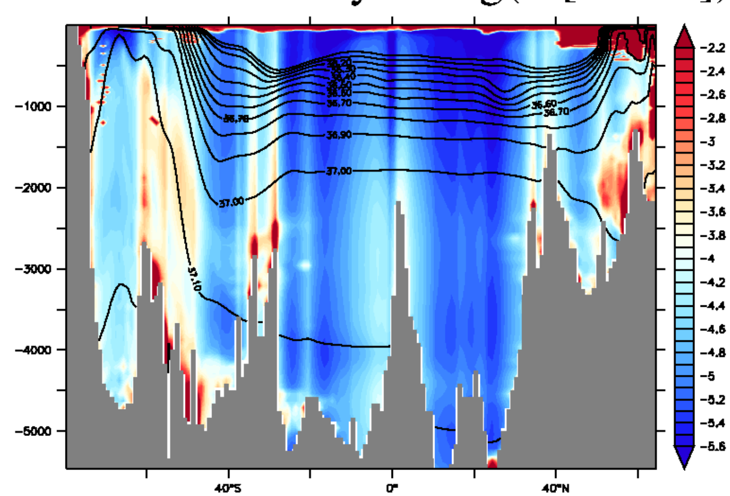
$$\frac{de_{iw}}{dt} = \frac{\partial}{\partial z} c_0 \tau_v \frac{\partial}{\partial z} c_0 e_{iw} + \nabla_h \cdot v_0 \tau_h \nabla_h v_0 e_{iw} - \epsilon_{iw}$$

- based on the integrated radiative transfer equation for internal waves
- closures for non-linear wave-wave interactions:
  - asymmetries in up- and downward prop. waves damped with time scale τ_v
  - dissipation of e_{iw} by $\epsilon_{iw} = \mu f\, e_{iw}^2 / c_\star^2$ as used in dissipation estimates from fine structure (e.g. Sun and Kunze, 1999)

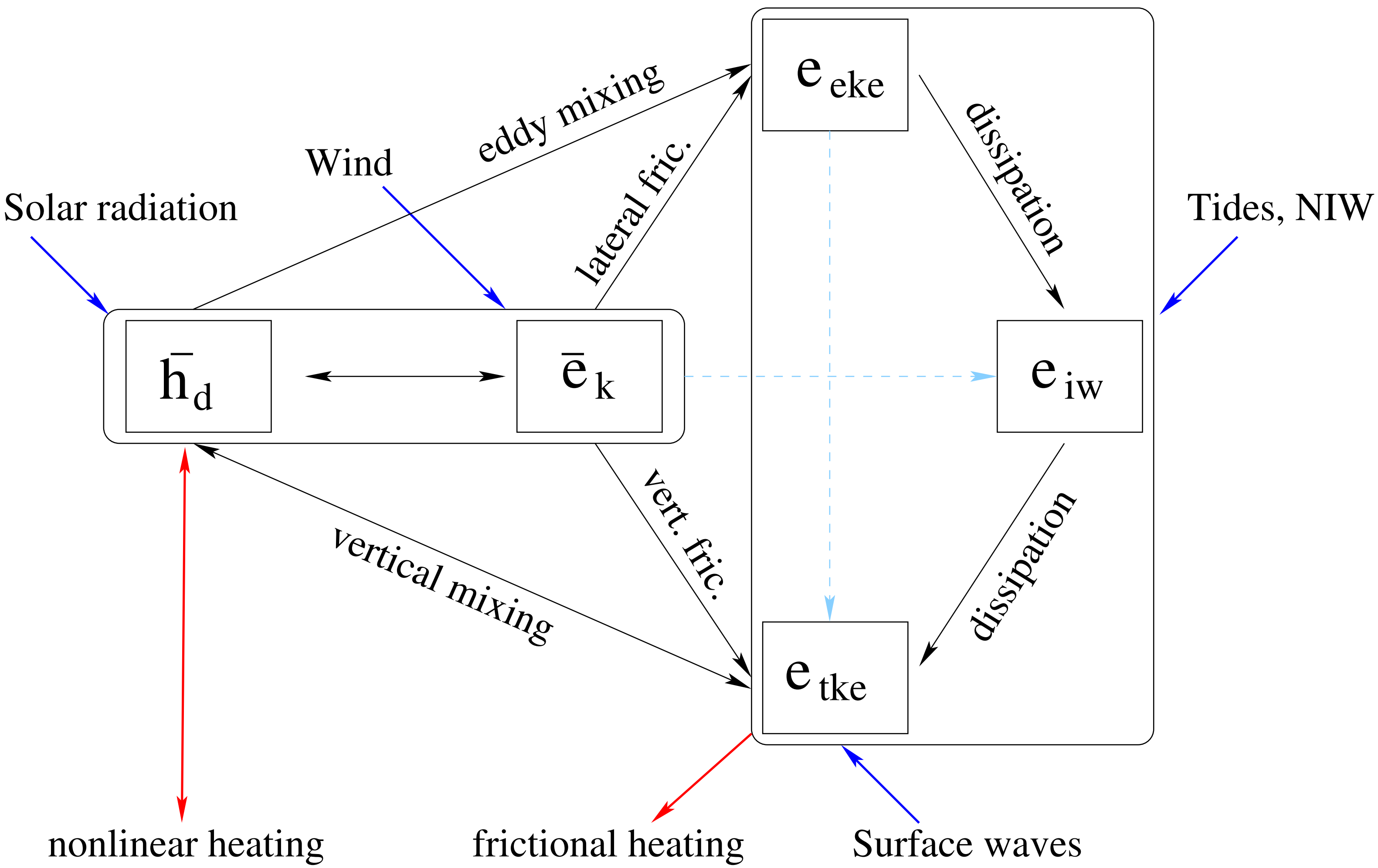
Tidal forcing in log(1/[W/m^2])



vertical diffusivity in log(1/[m^2/s])



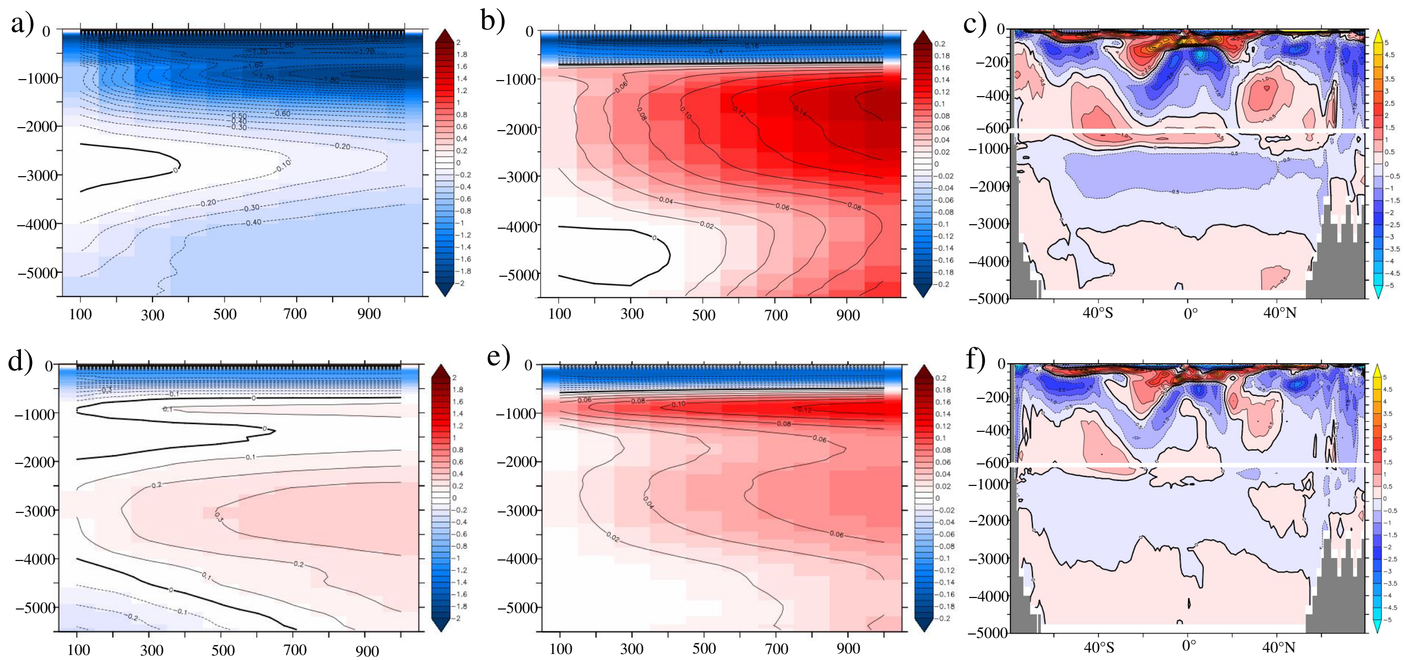
consistent energy cycle of an ocean model



- forcing by blue, dissipation by red arrows

consistent realistic global model: model biases

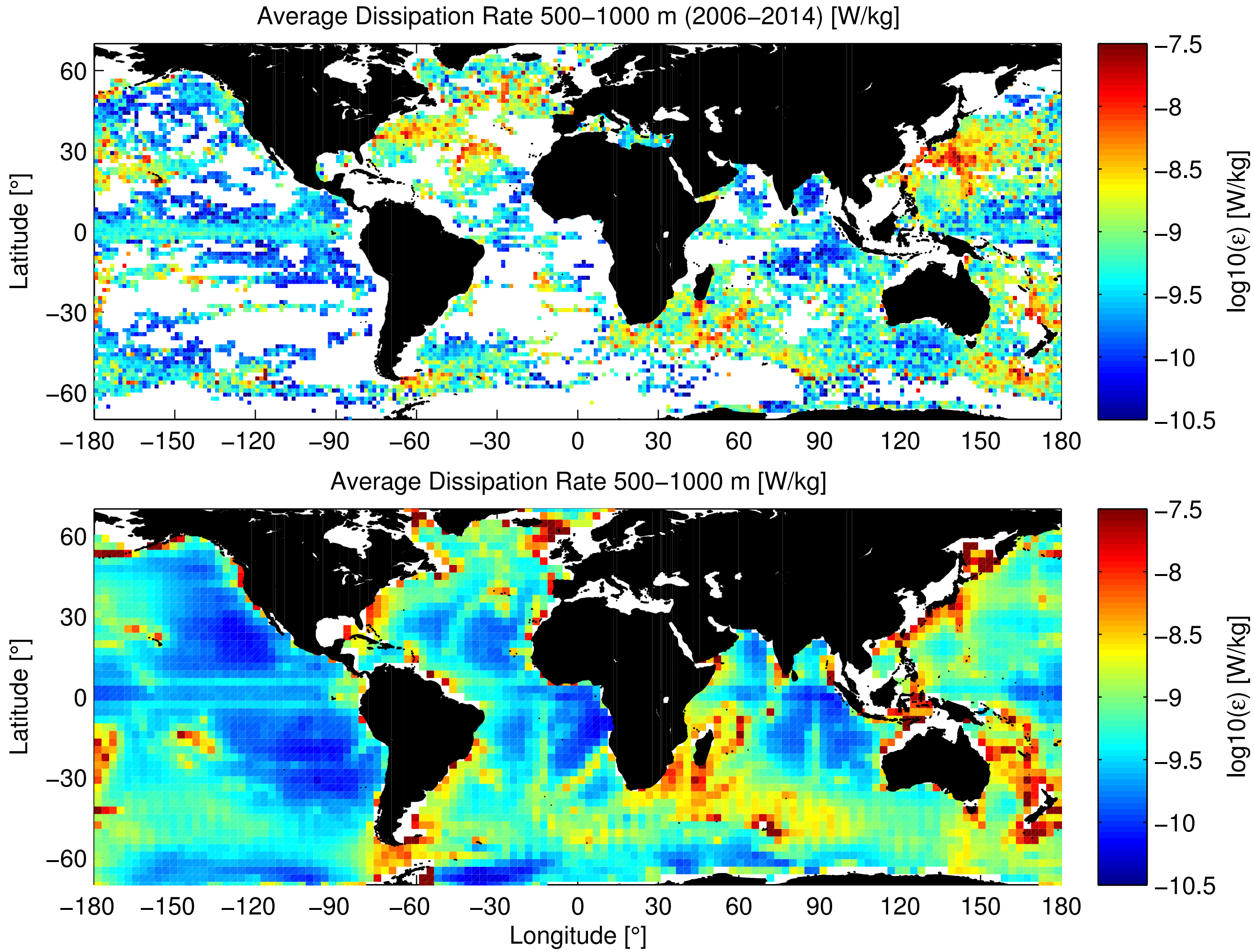
- consistent new numerical implementation (*wiki.zmaw.de/ifm/TO/pyOM2*)
- inconsistent model (upper row) and consistent model (lower row)



from Eden, Czeschel, Olbers (2014)

- a,d) horizontally averaged temperature bias (in $^o\,\mathrm{C}$)
- b,e) horizontally averaged salinity bias (in $\mathrm{g/kg}$)
- c,f) zonally averaged bias in N ($10^3\,\mathrm{s}^{-1}$)

consistent realistic global model: internal wave field

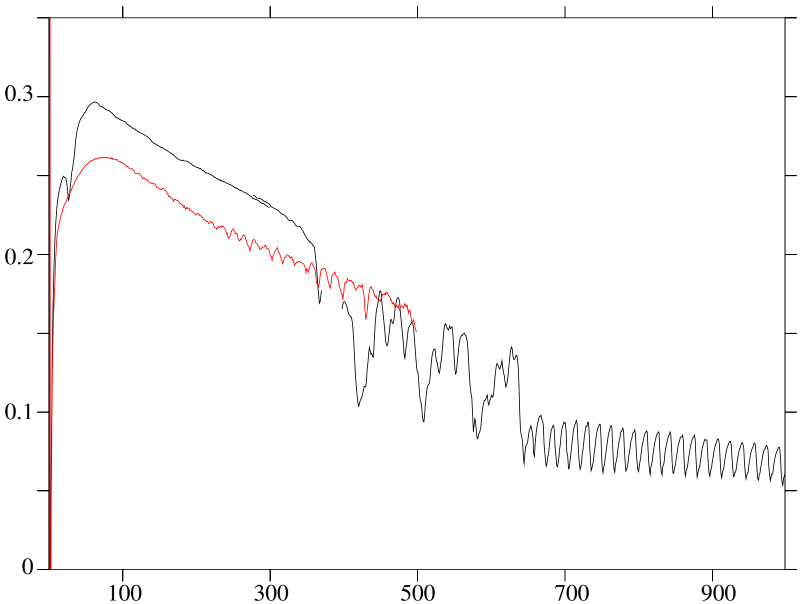


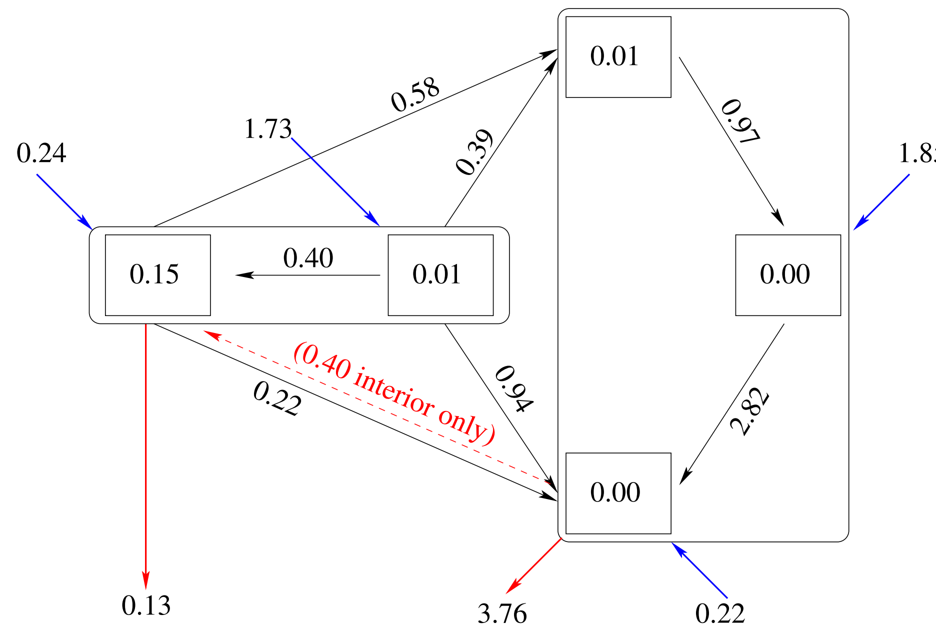
from Pollmann, Eden and Olbers (2015)

- upper: dissipation of e_{iw} from ARGO floats in $log_{10}(1/[\mathrm{W/kg}])$
- lower: dissipation of e_{iw} simulated by IDEMIX in model

consistent realistic global model: energy cycle

- total energy $\log_{10}(|\int \dot{e}_{tot} dV|/[\mathrm{W}])$ in $1^o \times 1^o$ (red) and $4^o \times 4^o$ global model





- transfer rates in TW, energy residual of only 800 W
- ca. $1.8\,\mathrm{TW}$ forcing from wind and tides, respectively
- ca. $1\,\mathrm{TW}$ e_{eke} dissipation, $3\,\mathrm{TW}$ e_{iw} dissipation
- ca. $0.4\,\mathrm{TW}$ forcing of $\bar{h}_d$ by interior mixing

meso-scale eddy dissipation

- dissipation of e_{eke} by $\epsilon_{eke} = e_{eke}^{3/2}/L$
- $1\,\mathrm{TW}$ dissipation of e_{eke} in the interior $\rightarrow$ too much mixing
- first ad hoc solution: 80% of ϵ_{eke} is injected to e_{tke} at the surface 20% of ϵ_{eke} at the bottom
- possible physical for dissipation of mechanism e_{eke} include
  - lee wave generation at bottom
  - forward energy cascade for small Ri (large Ro)
  - interior loss of balance
  - dissipation in bottom boundary layer
  - interaction at western boundaries
  - interaction with gravity waves (gravity wave drag)

lee wave generation at bottom

- geostrophic flow $\boldsymbol{u}$ over topography with spectrum $P(k)$
  $\rightarrow$ flow generates internal waves

- Bell (1975): energy flux into internal wave field is given by

$$F = \frac{\rho_0}{2\pi}|\boldsymbol{u}| \int_{|f|/|\boldsymbol{u}|}^{N/|\boldsymbol{u}|} dk\, P(k)\sqrt{N^2 - |\boldsymbol{u}|^2 k^2}\sqrt{|\boldsymbol{u}|^2 k^2 - f^2}$$

- with model topography spectrum from Goff and Jordan (1988)

$$P(k) \approx 2\overline{h^2_{rms}} k_0^{\mu-2} k^{-\mu+1}$$

  with $\mu \approx 3$ and parameter h^2_{rms} and k_0 estimated
  from seafloor spreading rates by Goff and Arbic (2009)
  or from depth sounding data

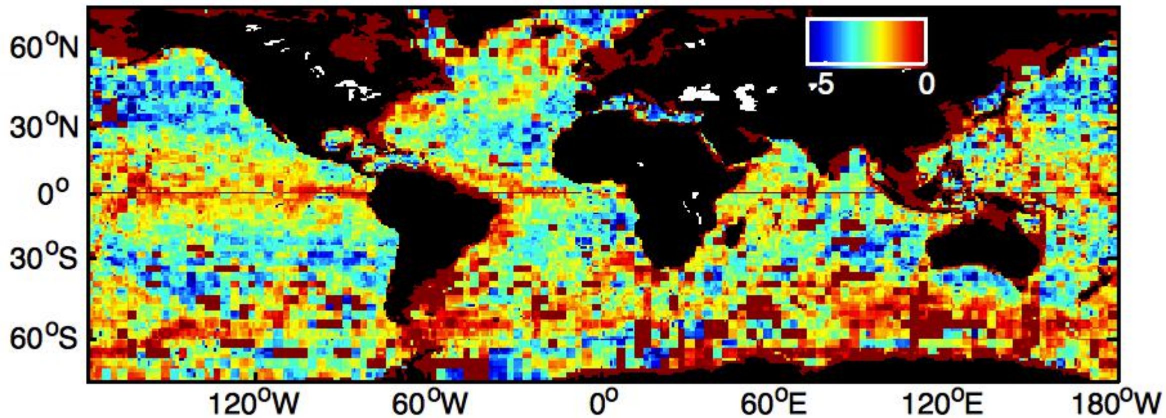
- Nikurashin and Ferrari (2010): flux can be simplified to

$$F = \frac{2}{\pi}\rho_0 \overline{h^2} \kappa_0^{3/2} N^{1/2} \left(\frac{3}{2} \left(\frac{N}{|f|} \right)^{1/2} - 2 \right) |\boldsymbol{u}|^{10/8} = r(x,y)\, e_{eke}^{5/4}$$

  which can be used in a model simulating e_{eke}

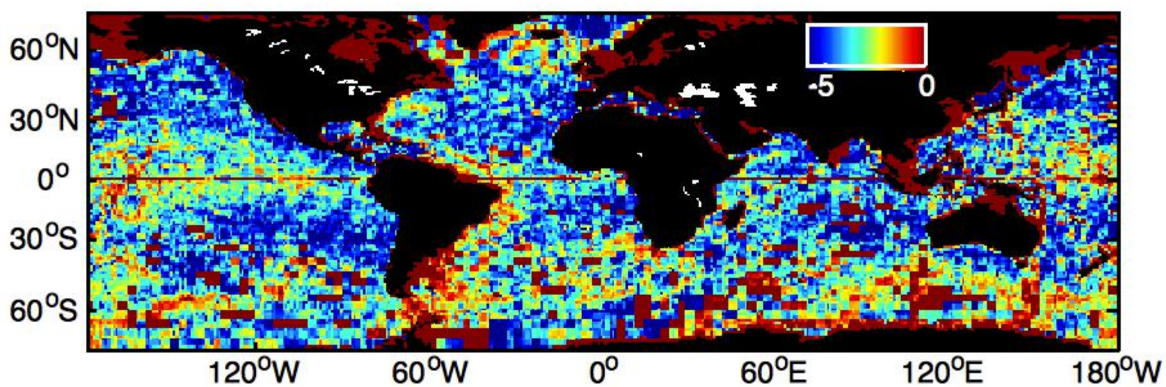
lee wave generation at bottom

- EKE and MKE from global eddying model (von Storch et al 2013)
- flux in $\log_{10} \mathrm{mW/m^2}$ from EKE using single beam data: 0.32 TW



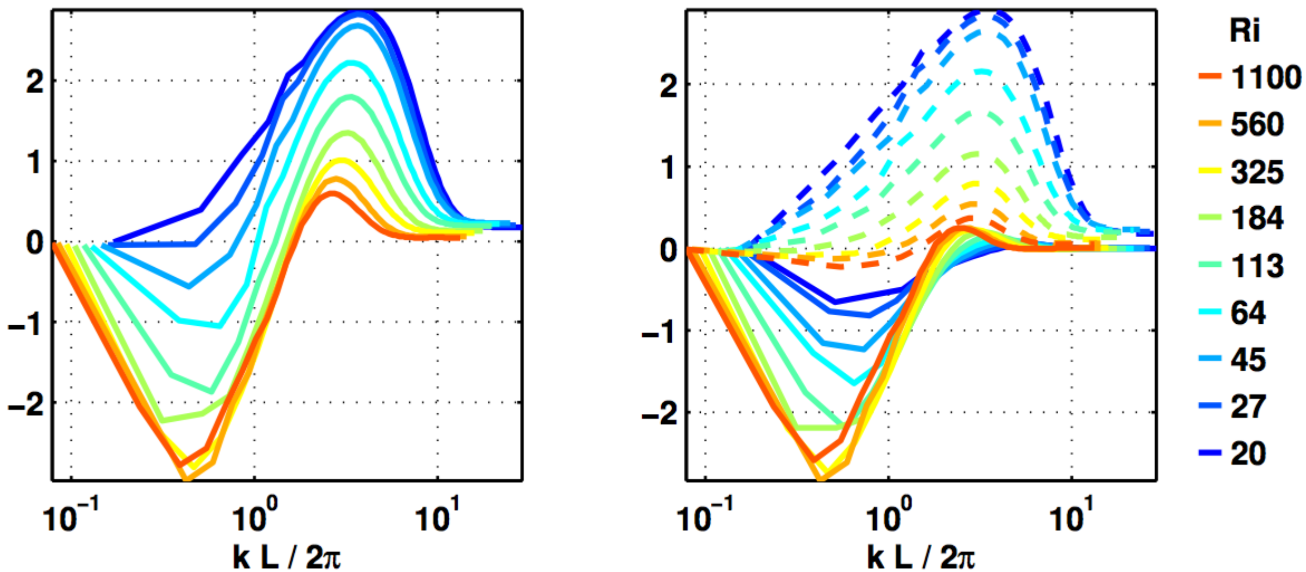
from Han and Eden (2015)

- flux from MKE 0.27 TW



forward energy cascade for small *Ri* (large *Ro*)

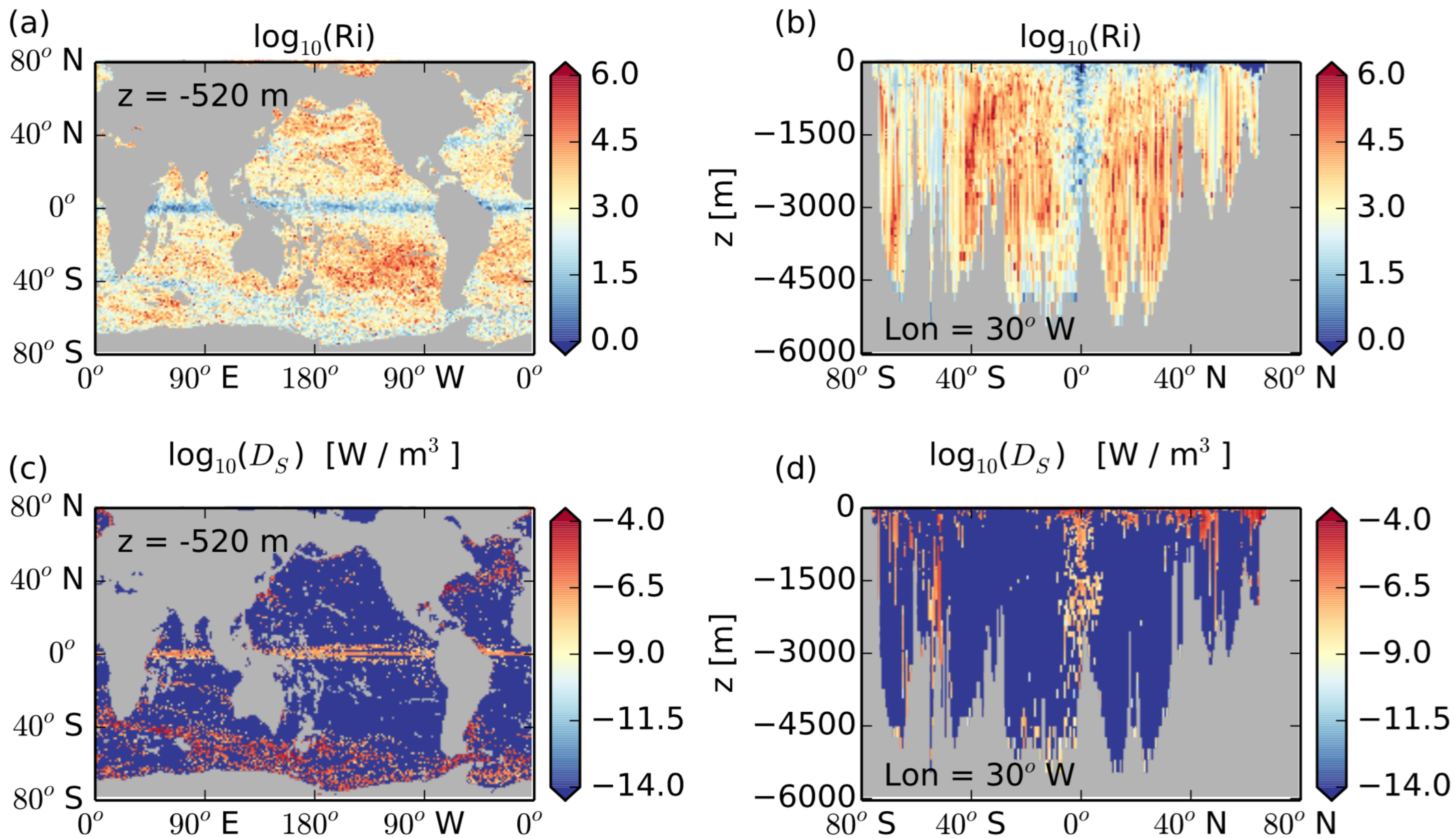
- idealized zonal channel simulation with baroclinic forcing for different $Ri = 20$ (large Ro) to $Ri = 1100$ (small Ro)
- kinetic energy fluxes in wavenumber space in $\mathrm{m}^2/\mathrm{s}^3$: full flux (left), flux due to rotational (QG) component (right, solid), residual flux (right dashed)



from Brüggemann and Eden (2015)

- inverse (forward) energy flux for rotational (residual) flux
- $\rightarrow$ parameterize interior eddy dissipation by $\epsilon_{eke} = r(Ri) e_{eke}$

forward energy cascade for small *Ri* (large *Ro*)



from Brüggemann and Eden (2015)

- a,b): $\log_{10} Ri$ in high resolution model by Storch et al (2012)
- c,d): diganosed dissipation rate $\epsilon_{eke} = r(Ri)e_{eke}$
- global integral of $0.26\,\mathrm{TW}$ excluding equator, half of it in the Southern Ocean

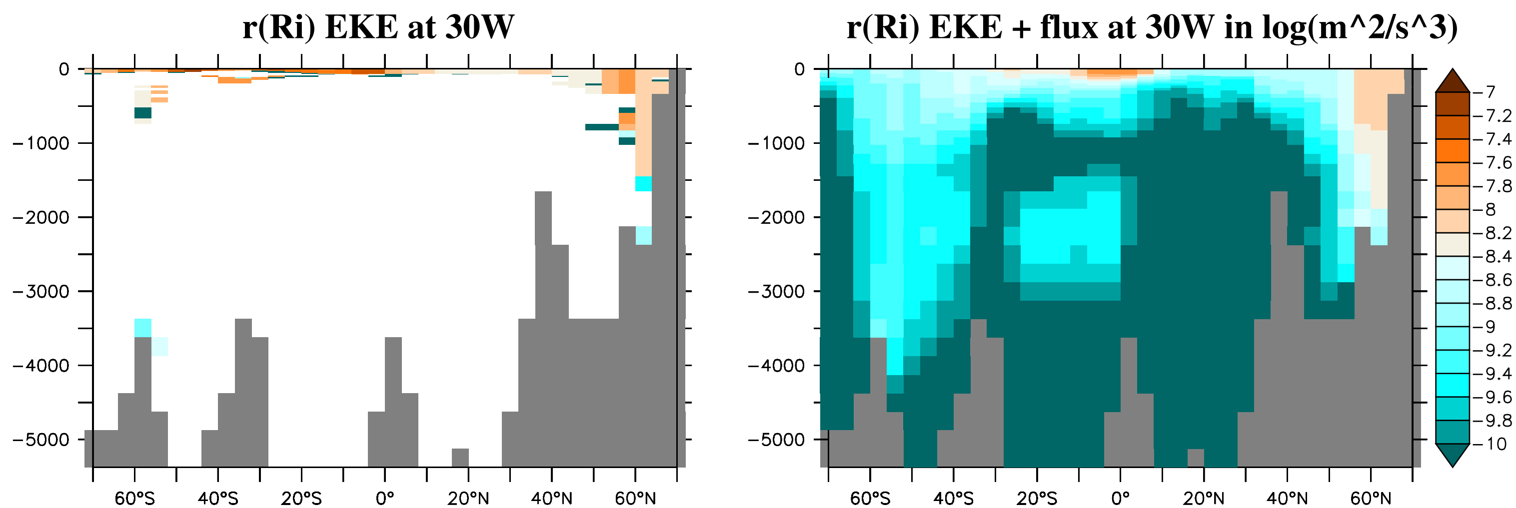
better closure for dissipation of balanced flow

$$\frac{De_{eke}}{Dt} = production + \nabla \cdot \boldsymbol{F} - r(Ri)e_{eke}$$

- with flux by lee waves leaving e_{eke} by $\boldsymbol{F} \cdot \boldsymbol{k}|_{z=-h} = F_{lee} = r'(x,y)\, e_{eke}^{5/4}$
- flux $\boldsymbol{F}$ connects region with strong dissipation (bottom, mixed layer) with regions of strong production: but $\boldsymbol{F}$ is not downgradient of e_{eke}
- parameterization for the combined effect of flux and dissipation

$$\nabla \cdot \boldsymbol{F} - r(Ri)e_{eke} \stackrel{!}{=} \nabla_h \cdot \boldsymbol{F}_h - F_{lee}\delta(z+h) - \frac{\int r(Ri)e_{eke}dz}{\int e_{eke}dz} e_{eke}$$

- vertical integral of both sides is identical



summary

- an energetically consistent ocean model made of
  - IDEMIX: internal wave dissipation and mixing (Olbers and Eden 2014, Eden and Olbers 2014)
  - meso-scale eddy closure (Eden and Greatbatch, 2008)
  - small-scale turbulent energy (Gaspar et al 1990)
- new numerical implementation with only a few $100\,\mathrm{W}$ residual globally
- biases in water masses are reduced in consistent model
- problems are:
  - eddy dissipation $\rightarrow$ lee wave generation and $r(Ri)$ damping
  - lateral or biharmonic friction
  - ...