XBT Bias and Fall-rate Workshop
August 25-27, 2010
Klima Campus, The University of Hamburg, Hamburg, Germany

A numerical model for free-fall type sensor in order to check fall-rate of probe

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Remarks:
This study was made in 1990 (twenty years ago!) and is not completed yet.

Motivation of this study

Fall-rate equation for XBT probe: $z = at + bt^2$

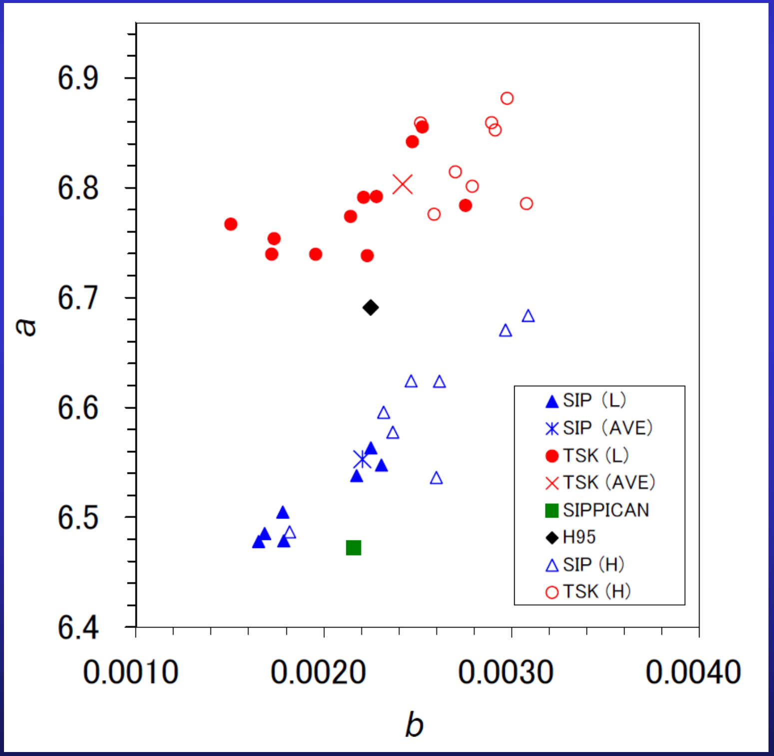
z: depth of probe in meter
t: elapsed time from the hit at the sea surface in second
a and *b*: constants to be determined empirically

The results of comparison experiment between XBT vs. CTD are plotted on the '*a-b*' plane.

In general, obtained *a-b* pairs for individual probe widely scatter on the *a-b* plane (see Fig).

What is the reason responsible for this scatter? What kinds of caution should XBT manufacturers pay their attention in produc-tion of XBT probe to keep its specification in accuracy of depth?

In order to clarify these points, numerical model would be useful.



Kizu et al. (2010)
Submitted to Ocean Science

Model for free-fall type sensors

Basic equations for free-fall type sensors (so-called 'Rocket launching model')

$$(M+m)\frac{d}{dt}s = \{(M+m) - \underbrace{\rho(V+v)}_{①}\}g - \underbrace{(k+dk)}_{②}s^2$$

equation of motion

$$\frac{d}{dt}(M+m) = s\frac{m}{L}$$

temporal change of probe weight

① buoyancy

② effect of drag

z (depth) should be inserted here.

M: weight of the probe except for wire (kg)

V: volume of the probe except for wire (m^3)

m: weight of the wire installed in probe (kg)

v: volume of the wire installed in probe (m^3)

L: initial length of the wire in the probe(m)

ρ : water density (kg/m^3)

s: free-fall velocity of the probe (m/s)

g: acceleration of gravity (m/s^2)

k: effective drag coefficient of the probe proportional to square of velocity (kg/m^4)

dk: variable component of k linearly depending on the probe depth (kg/m^3)

k and dk : unknown parameters to be determined

Model for free-fall type sensors

Introduction of new variables:

$$M_0 = (M + m) - \rho(V + v)$$ total weight of the probe in the water

$$m_0 = \frac{1}{L}(m - \rho v)$$ weight of the wire per unit length in the water

Temporal change of weight and equation of motion

$$\frac{d}{dt}M_0 = s m_0$$

$$\frac{d}{dt}s = \left\{1 - \frac{\rho(V + v)}{M_0}\right\}g - \frac{(k + zdk)s^2}{M_0}$$

numerical integration:
Forth-order Runge-Kutta method

Initial conditions for total weight of the probe and initial velocity

$$M_0 = M_0(0) \quad at \quad t = 0$$

$$s = s(0) \quad at \quad t = 0$$

Calculation of depth using simulated velocity

$$z = \int_0^t s\,dt$$

Simpson’s method

Determination of ‘a’ and ‘b’ using simulated time-depth data

The ‘*a*’ and ‘*b*’ are obtained by the least square method using simulated time-depth data from 10 seconds (about 70m) to 100 seconds (about 750m).

Standard values of M_0 and m_0 : T-7 probe

Standard values (information provided by TSK)

M_0 = 0.58 kg : total weight of the probe in the water

m_0 = 0.0001 kg/m : weight of the wire per unit length in the water

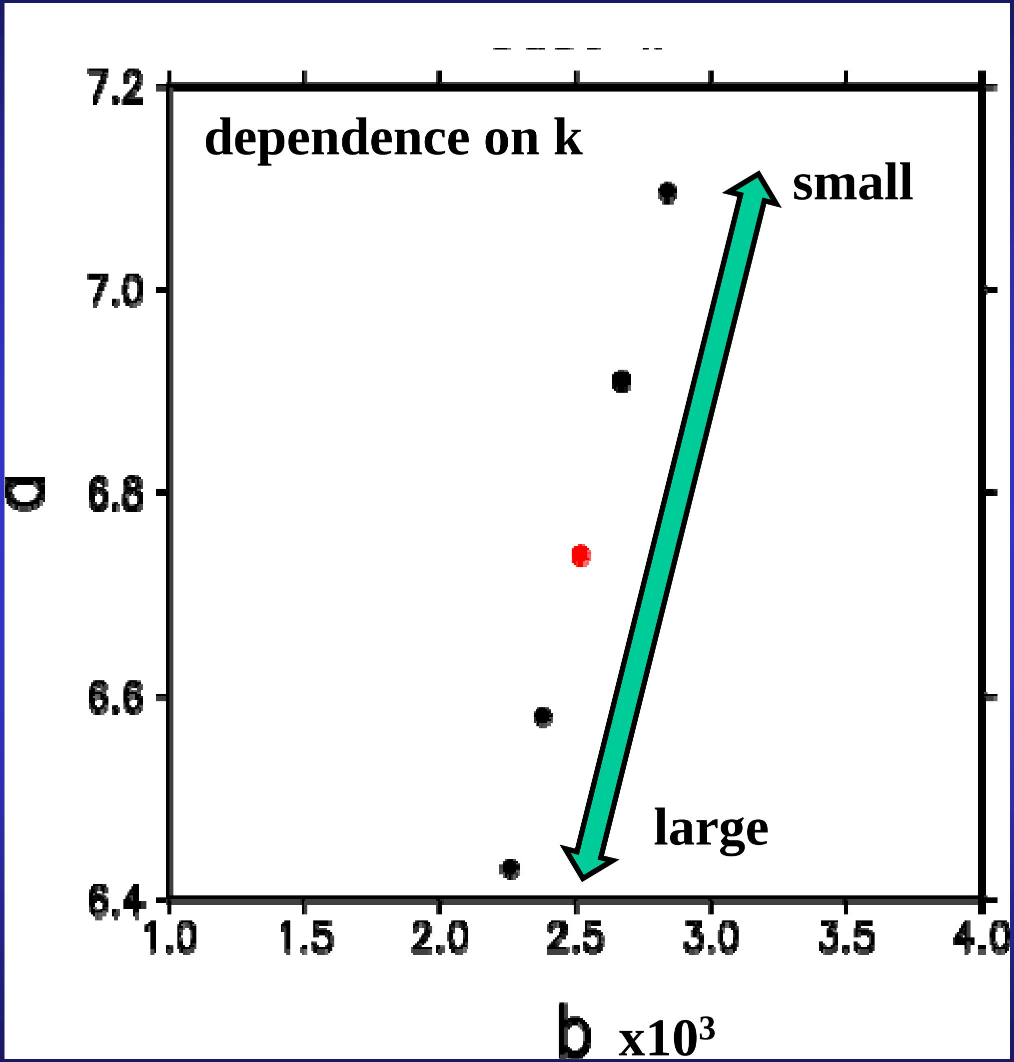
Determination of drag coefficients: k and dk

First of all, we have to set the plausible values of k and dk.

In order to obtain the plausible values for k and dk, 'try-and-error' calculations are made using various values of k and dk.

Figure shows '*a-b*' dependence on k for fixed dk (0.00001).

As k increases, then both *a* and *b* become larger.



k = 0.109350, --, 0.121500, --, 0.133650

Determination of drag coefficients: k and dk (continued)

Figure shows '*a-b*' dependence on dk for three kinds of k (0.115425, 0.121500, 0.127575).

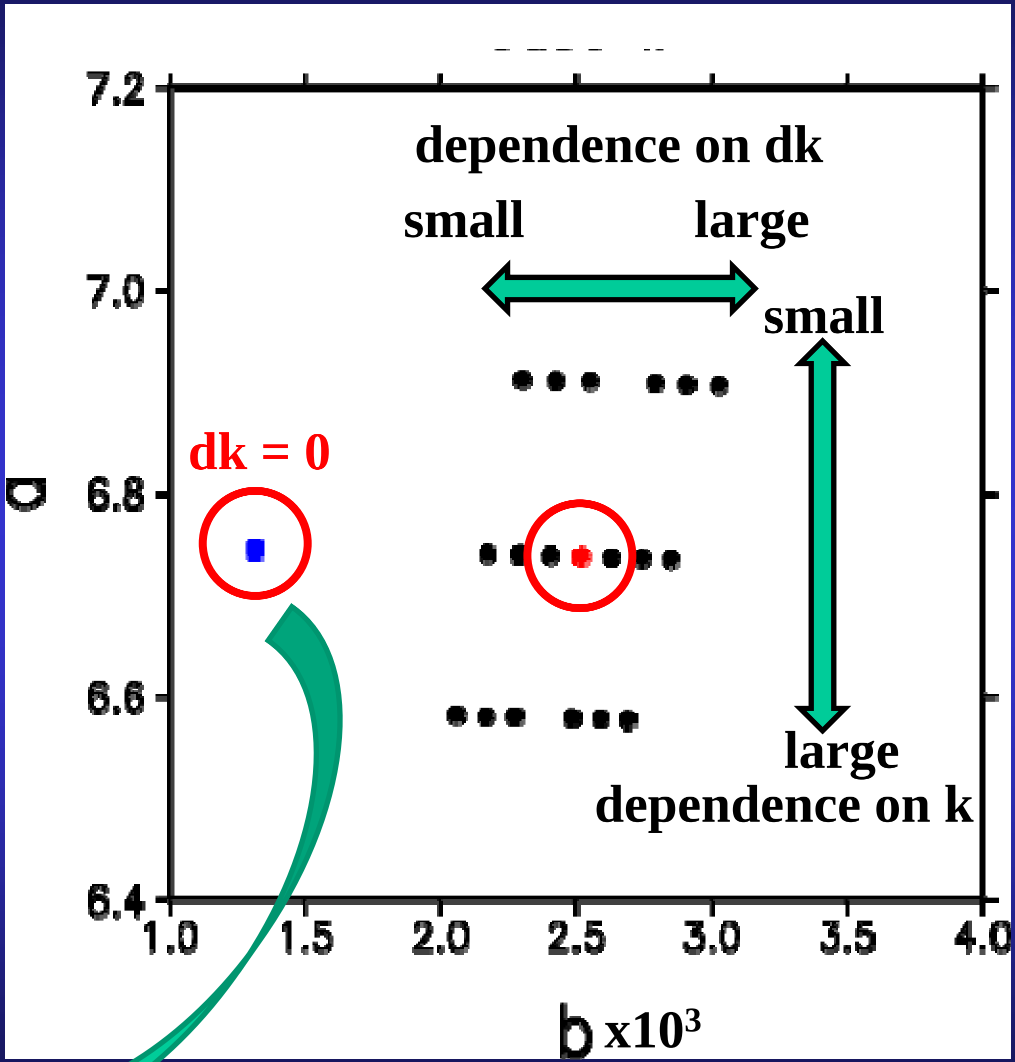
As dk increases, then *b* becomes larger, but *a* almost does not change.

Based on these results, we set
k = 0.121500 and dk = 0.00001.

These drag coefficients give the fall-rate equation,

$$z = 6.73846t - 2.51281 \times 10^{-3} t^2$$

This equation is regarded as the prototype (reference) fall-rate equation (red point in the figure) .



k = 0.115425, 0.121500, 0.127575
dk = 0.000007, -, -, -, -, -, 0.000013

When dk = 0, we can not obtain plausible value for *b*.

Dependence on total weight of the probe: M_0

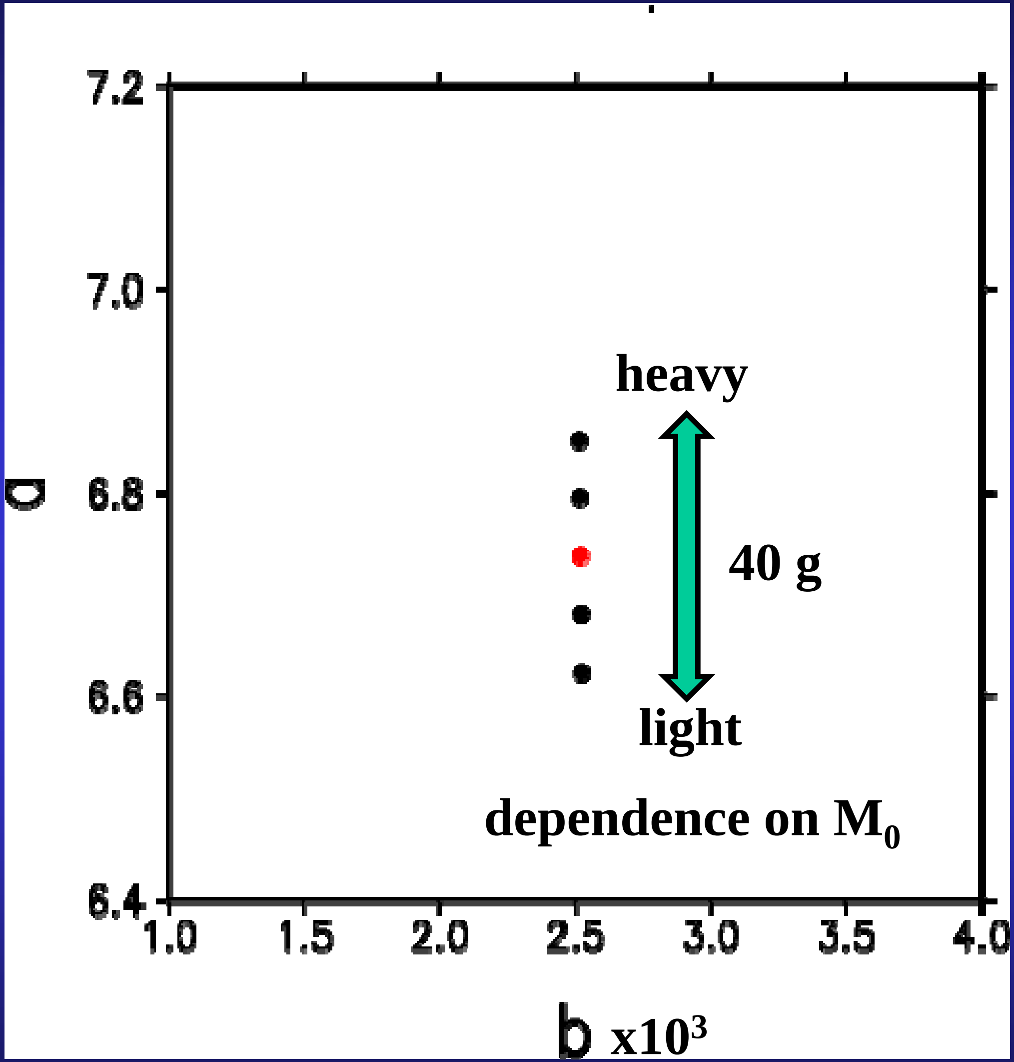
Since we set the prototype fall-rate equation, we can estimate effects of various parameters.

First, we estimate effect of probe weight (M_0) on *a*-*b* constants.

Figure shows *a*-*b* dependence on M_0.

As M_0 increases, then *a* becomes larger, but *b* does not change.

Roughly speaking , change of 10g in M_0 gives change of 0.06 in *a*.



M_0 = 0.56, 0.57, 0.58, 0.59, 0.60 (kg)

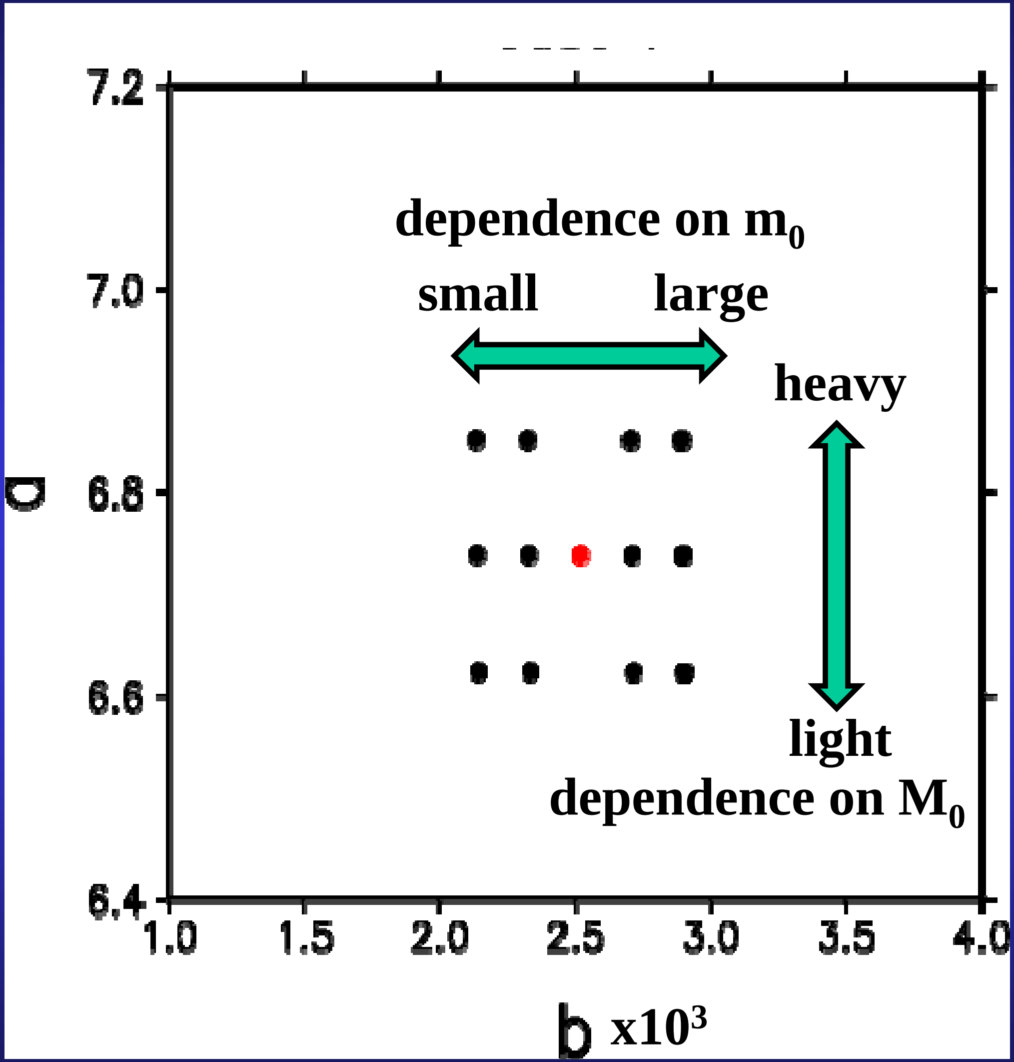
Dependence on wire weight (per unit meter): m_0

Next, we check effect of wire weight (m_0). Change of m_0 may come from change of wire thickness and thickness of enamel coating to wire.

Figure shows *a-b* dependence on m_0 for three fixed M_0.

As m_0 increases, then *b* becomes larger, but *a* does not change.

Roughly speaking , change of 0.00001 kg/m in m_0 gives change of 0.2x10^{-3} in *b*.



M_0 = 0.56, 0.58, 0.60
m_0 = 0.00008, -, 0.00010, -, 0.00012

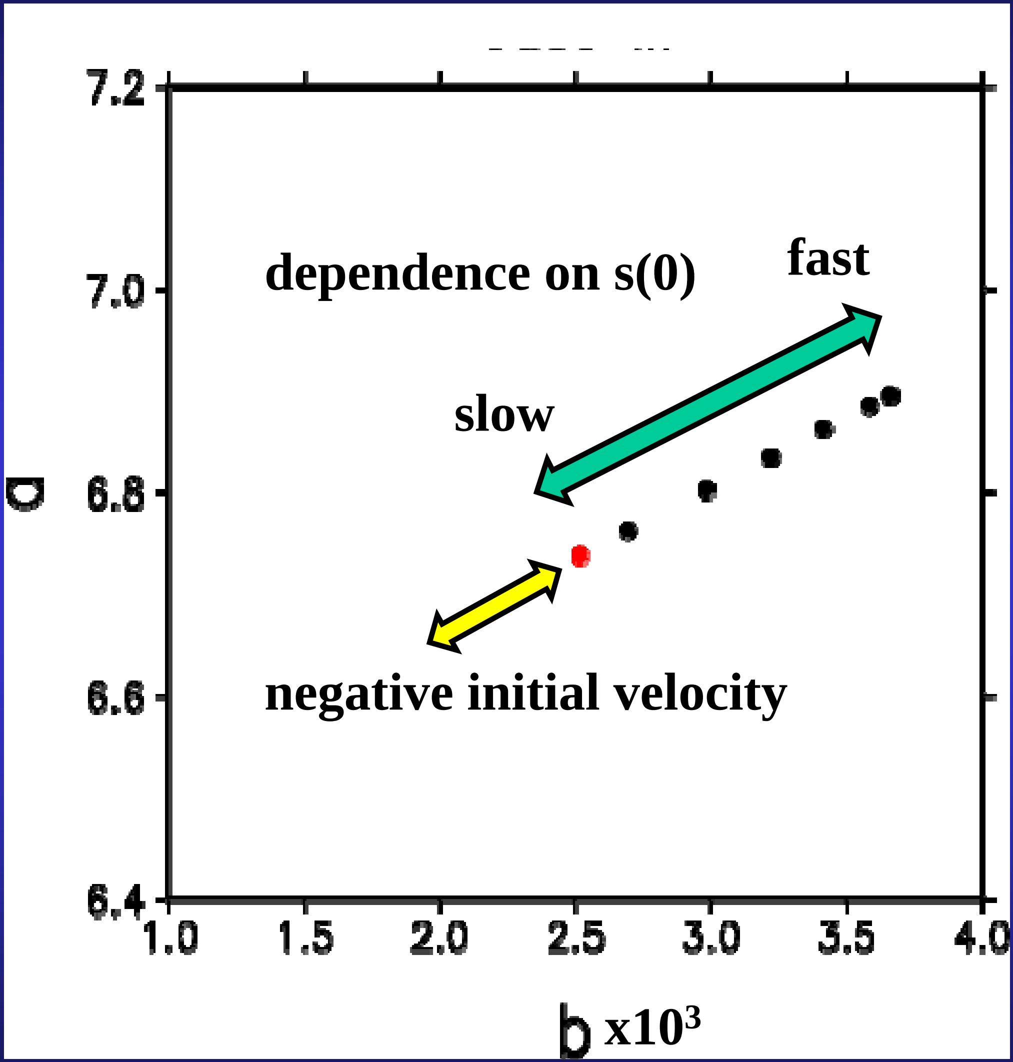
Dependence on initial velocity: s(0)

Sometimes XBTs are dropped from merchant vessels. So, XBT probe may have some initial velocity at the sea surface (s(0)).

Figure shows *a-b* dependence on s(0) (0 to 10m/s).

Interestingly, s(0) gives change of both *a* and *b*. As s(0) increases, then both *a* and *b* becomes larger.

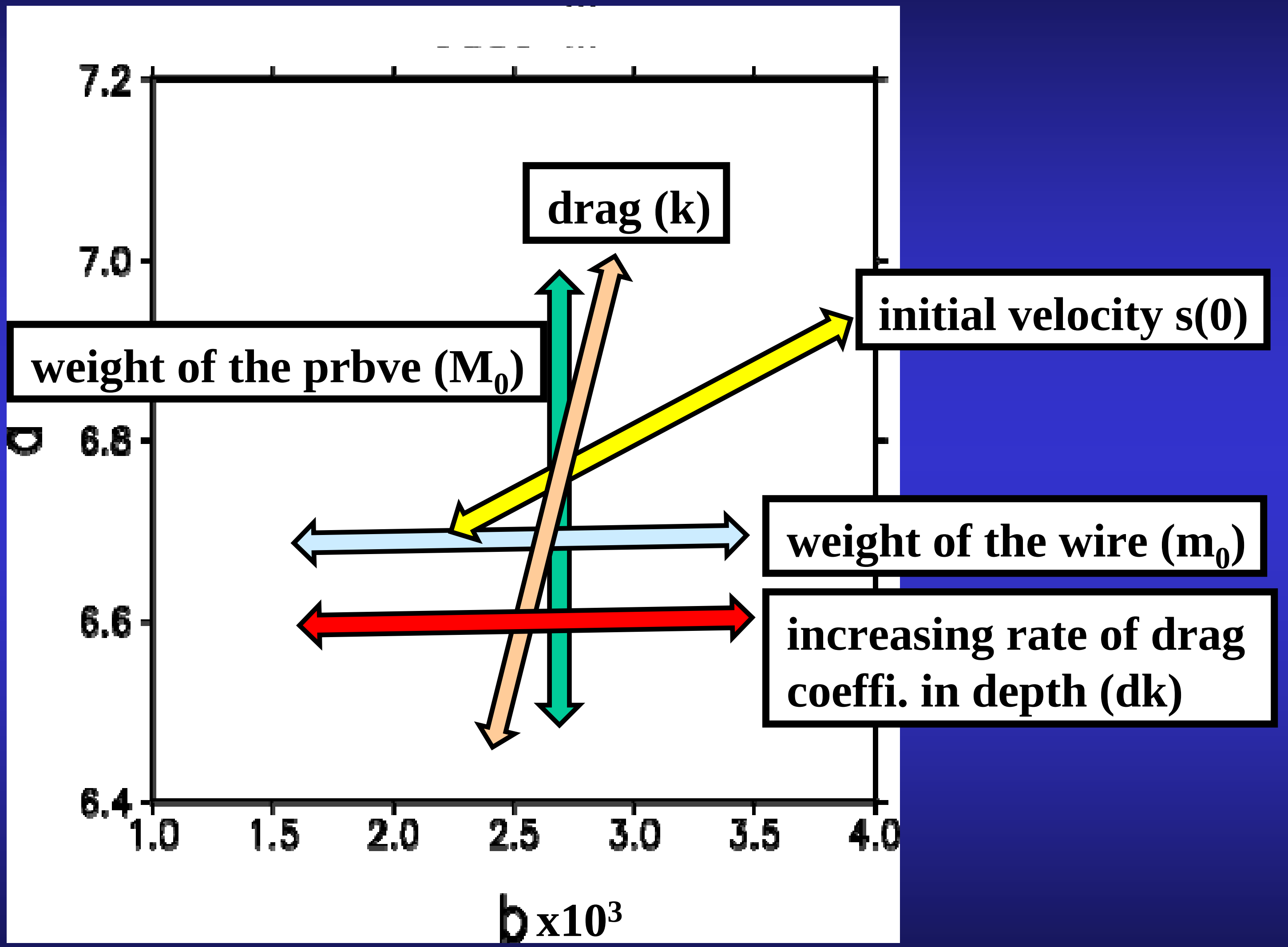
In reality, 'posture' of XBT probe when it hits the sea surface, is very important. So, we had better consider the situation of negative initial velocity, which may reflect the posture of XBT probe.



s(0) = 0, 1, 3, 5, 7, 9, 10 (m/s)

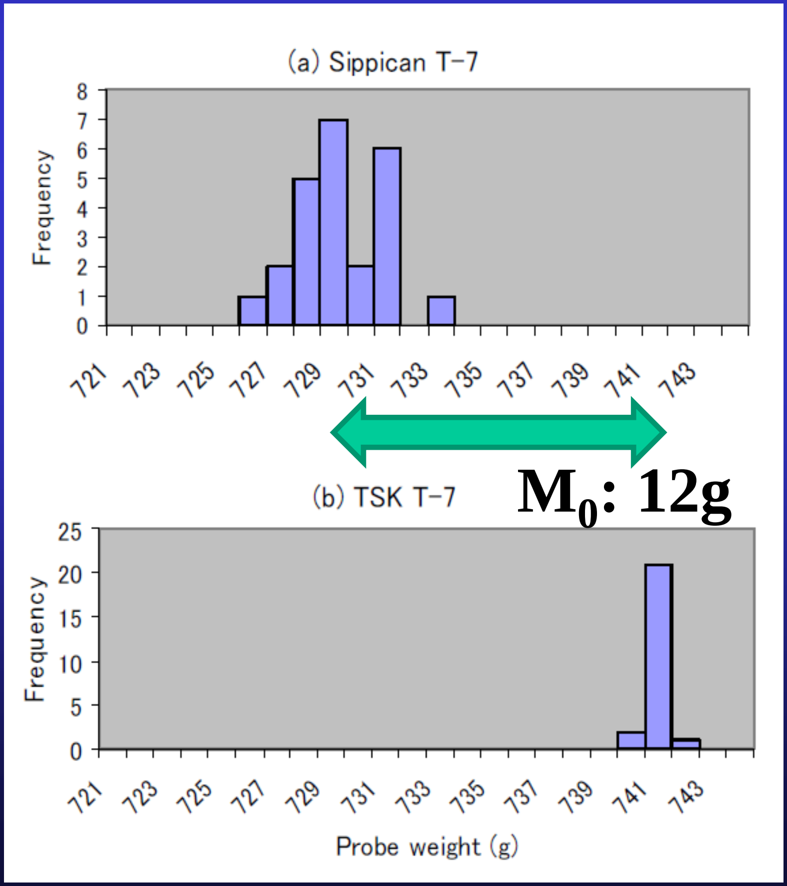
*** 'Posture' means the incident angle of the probe.**

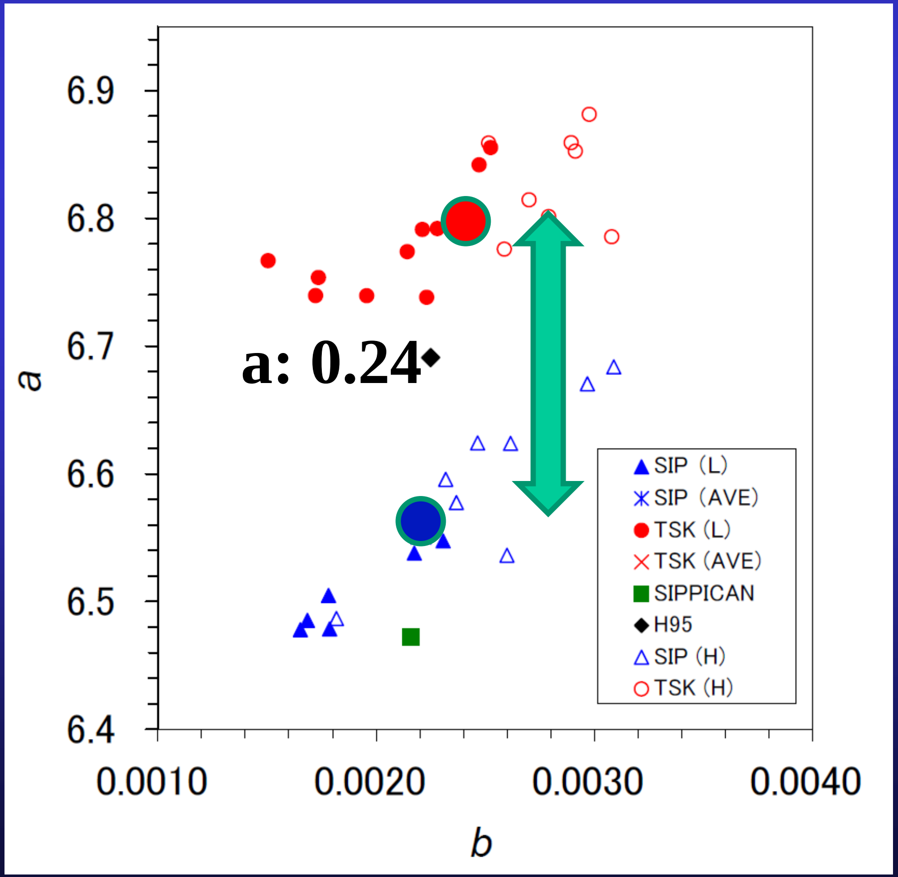
Summary of a-b dependence on various parameters



Interpretation of difference between Sippican and TSK probes

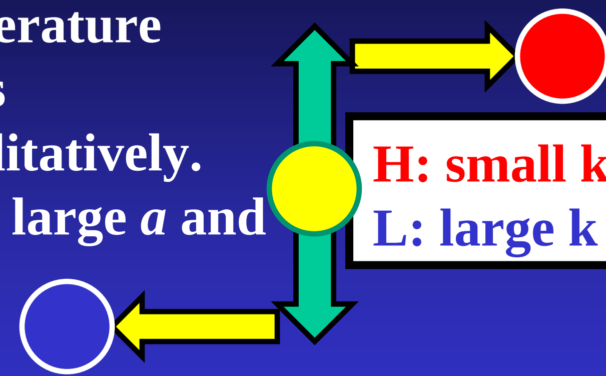
My model says that change of 10g in M_0 gives change of 0.06 in *a*.
So, difference of 0.24 in *a* can not be explained by difference of weight only. Other effect might influence on fall rate. I suppose the difference in shape cause difference effective drag coefficient (k) , and this different k gives different pairs of *a* and *b* between Sippican and TSK probes,



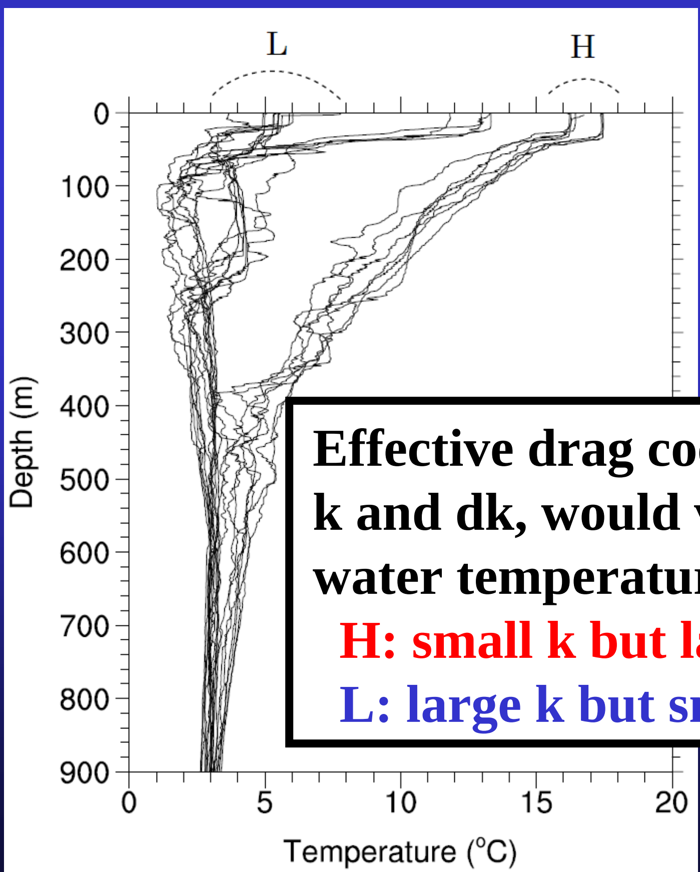


Interpretation of dependence on temperature profiles

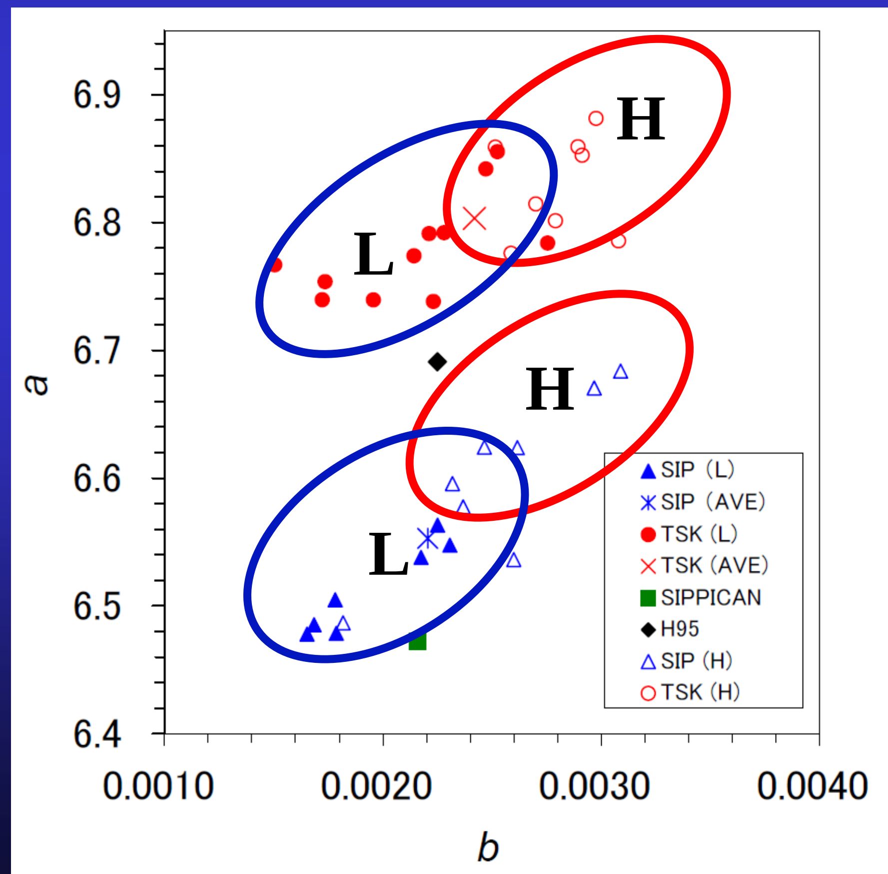
We can observe *a*-*b* dependence on temperature profiles (see figures) and can interpret its dependence based our model results qualitatively. H group (small k but large dk) may have large *a* and *b*, while L group (large k but small dk) may have small *a* and *b*.



H: small k but large dk
L: large k but small dk

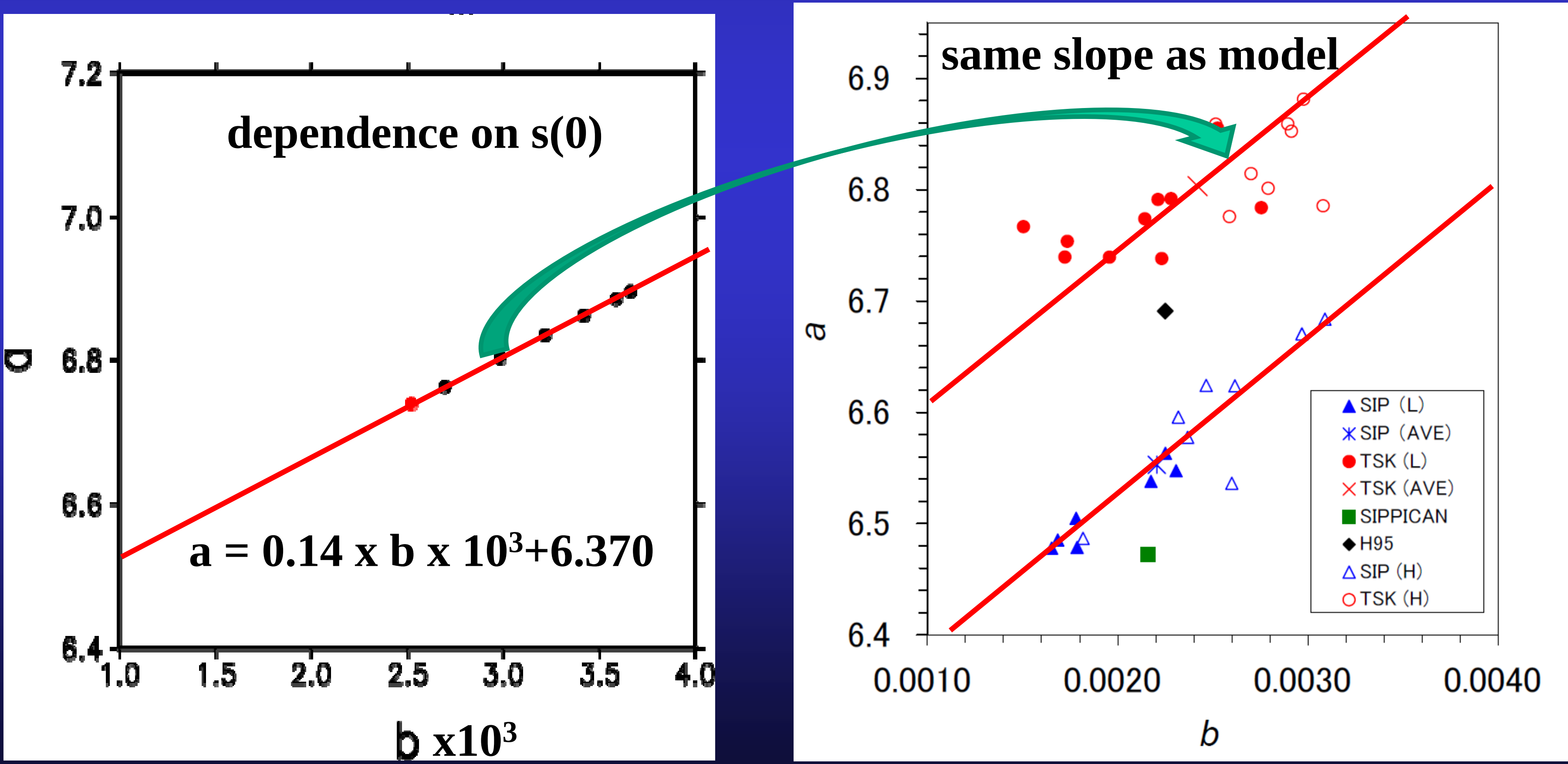


Effective drag coefficients, k and dk, would vary with water temperature.
H: small k but large dk
L: large k but small dk



Interpretation of scatter for individual a-b pairs

The scatter of *a-b* pairs for individual comparison data might be interpreted from the dependence on initial velocity of XBT probe (s(0)) at the sea surface. Actually, *a-b* pairs of Kizu et al. (2010) are distributed along the line estimated to simulated results for the s(0) dependence (see figures below).



Next step

Extension of this model

1. Inclusion of the effect of air bubbles trapped in the XBT probe
2. Inclusion of the effect of surface gravity waves
   (existence of vertical velocity)

Validation of this model

We have a plan to validate this model through XBT/CTD comparison experiment in the sea using artificially controlled XBT probes in weight (lighter and heavier probes).

Concluding remarks

The developed numerical model is so simple, but I believe the model can provide some useful suggestions concerning the degrees of allowable scatter of XBT probes to XBT manufacturers.

e.g., scatter of the weight of XBT probe in the water should be within +/- 2 g.

scatter of the weight of wire in the water should be within +/- 0.000001 g/m.

the prove shape is critical one, since it drastically changes the magnitude of effective drag. The probe shape should keep as it is absolutely.

etc.

I guess XBT probes (shape and weight etc.) have been changing in time, and therefore fall rate have been changing in time. We had better recommend to XBT manufacturers that XBT probes should not be changed from any kinds of viewpoints.