Dynamical systems and turbulence: tipping points, transients, and noise

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Tipping elements in the Earth's climate system

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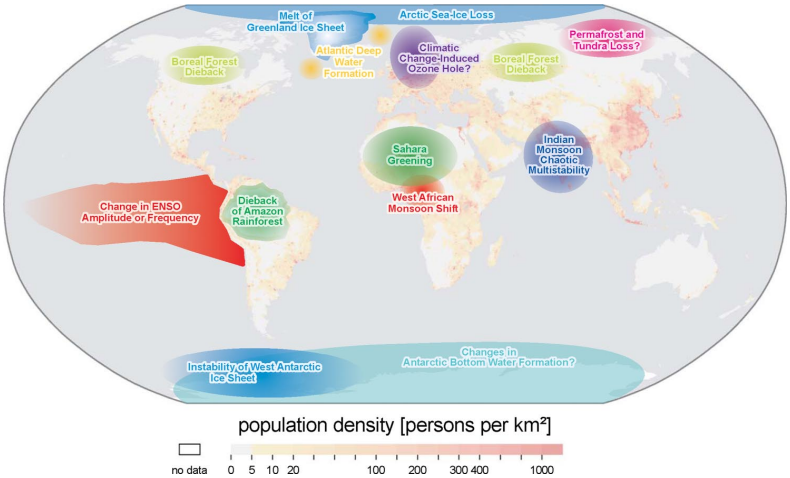

Lenton et al, PNAS **105**, 1786-93 (2008)

Table 1. Policy-relevant potential future tipping elements in the climate system and (below the empty line) candidates that we considered but failed to make the short list*

Tipping element	Feature of system, F (direction of change)	Control parameter(s), ρ	Critical value(s),† ρ_{crit}	Global warming†‡	Transition timescale,† T	Key impacts
Arctic summer sea-ice	Areal extent (−)	Local ΔT_{air}, ocean heat transport	Unidentified§	+0.5–2°C	~10 yr (rapid)	Amplified warming, ecosystem change
Greenland ice sheet (GIS)	Ice volume (−)	Local ΔT_{air}	+~3°C	+1–2°C	>300 yr (slow)	Sea level +2–7 m
West Antarctic ice sheet (WAIS)	Ice volume (−)	Local ΔT_{air}, or less ΔT_{ocean}	+~5–8°C	+3–5°C	>300 yr (slow)	Sea level +5 m
Atlantic thermohaline circulation (THC)	Overturning (−)	Freshwater input to N Atlantic	+0.1–0.5 Sv	+3–5°C	~100 yr (gradual)	Regional cooling, sea level, ITCZ shift
El Niño–Southern Oscillation (ENSO)	Amplitude (+)	Thermocline depth, sharpness in EEP	Unidentified§	+3–6°C	~100 yr (gradual)	Drought in SE Asia and elsewhere
Indian summer monsoon (ISM)	Rainfall (−)	Planetary albedo over India	0.5	N/A	~1 yr (rapid)	Drought, decreased carrying capacity
Sahara/Sahel and West African monsoon (WAM)	Vegetation fraction (+)	Precipitation	100 mm/yr	+3–5°C	~10 yr (rapid)	Increased carrying capacity
Amazon rainforest	Tree fraction (−)	Precipitation, dry season length	1,100 mm/yr	+3–4°C	~50 yr (gradual)	Biodiversity loss, decreased rainfall
Boreal forest	Tree fraction (−)	Local ΔT_{air}	+~7°C	+3–5°C	~50 yr (gradual)	Biome switch
Antarctic Bottom Water (AABW)*	Formation (−)	Precipitation–Evaporation	+100 mm/yr	Unclear¶	~100 yr (gradual)	Ocean circulation, carbon storage
Tundra*	Tree fraction (+)	Growing degree days above zero	Missing‖	—	~100 yr (gradual)	Amplified warming, biome switch
Permafrost*	Volume (−)	$\Delta T_{permafrost}$	Missing‖	—	<100 yr (gradual)	CH_4 and CO_2 release
Marine methane hydrates*	Hydrate volume (−)	$\Delta T_{sediment}$	Unidentified§	Unclear¶	10^3 to 10^5 yr ($>T_E$)	Amplified global warming
Ocean anoxia*	Ocean anoxia (+)	Phosphorus input to ocean	+~20%	Unclear¶	~10^4 yr ($>T_E$)	Marine mass extinction
Arctic ozone*	Column depth (−)	Polar stratospheric cloud formation	195 K	Unclear¶	<1 yr (rapid)	Increased UV at surface

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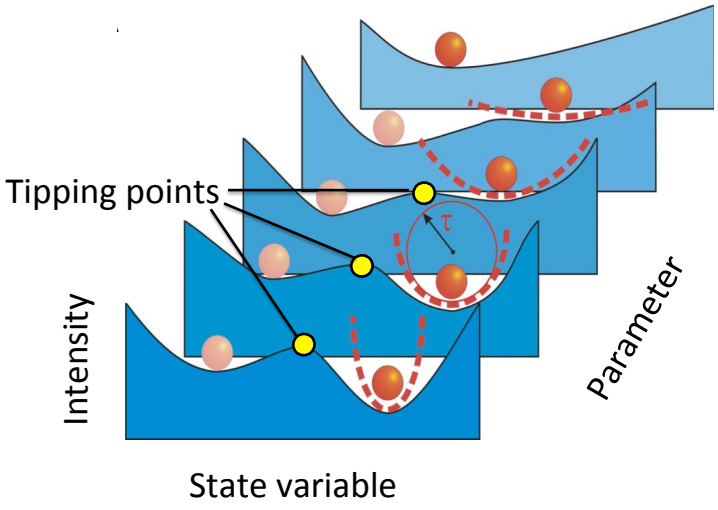
Lenton et al, PNAS **105**, 1786-93 (2008)



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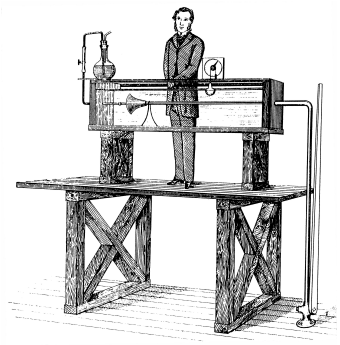
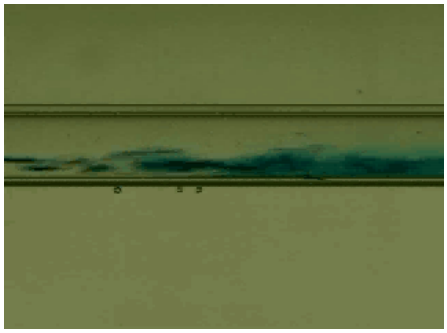
Lenton et al, PNAS **105**, 1786-93 (2008)

Reynolds' experiment (1883)

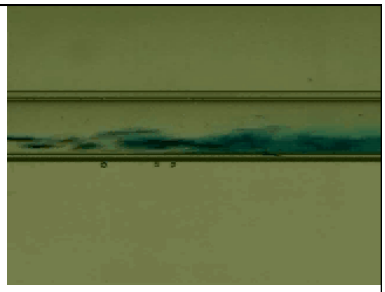




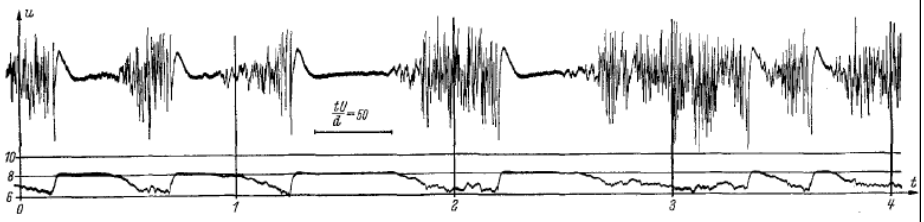
O. Reynolds, Phil Trans R Soc 1883

G Homsy et al, Fluid Dynamics CD

Rotta 1956



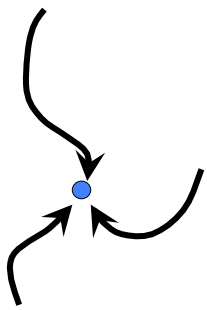
- Spatio-temporal intermittency

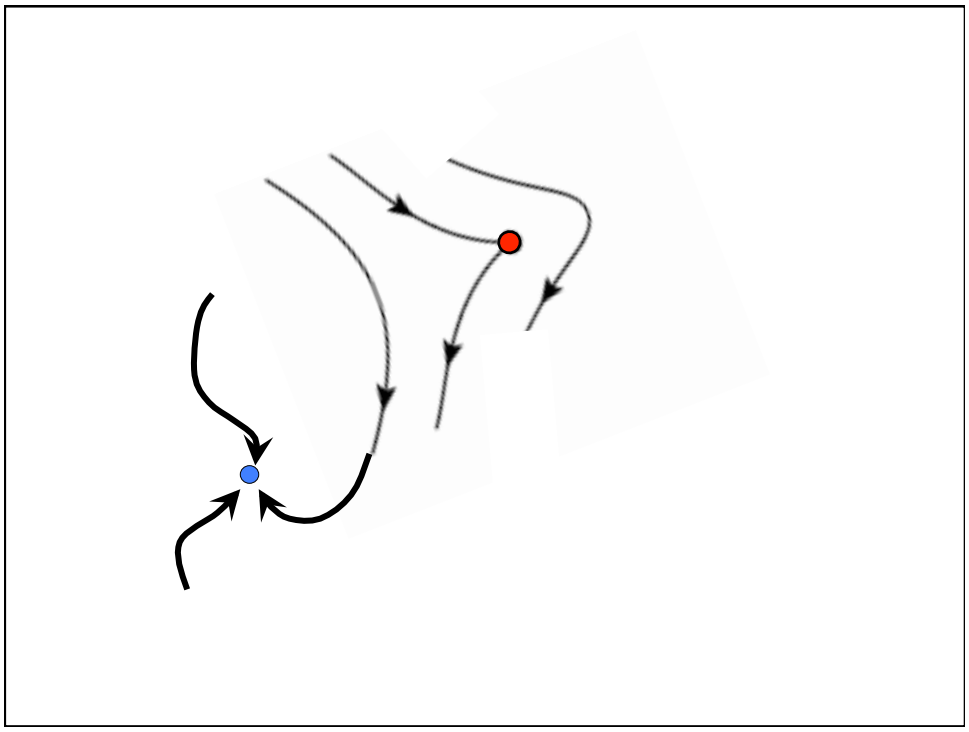


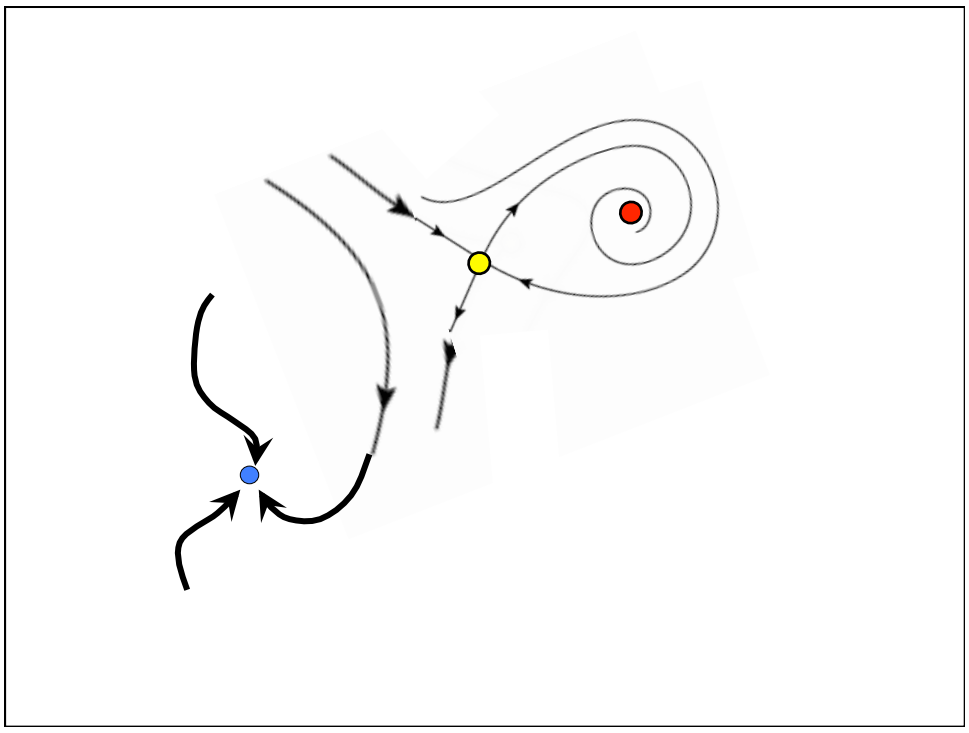
- Intrinsic switches between turbulent and laminar flow

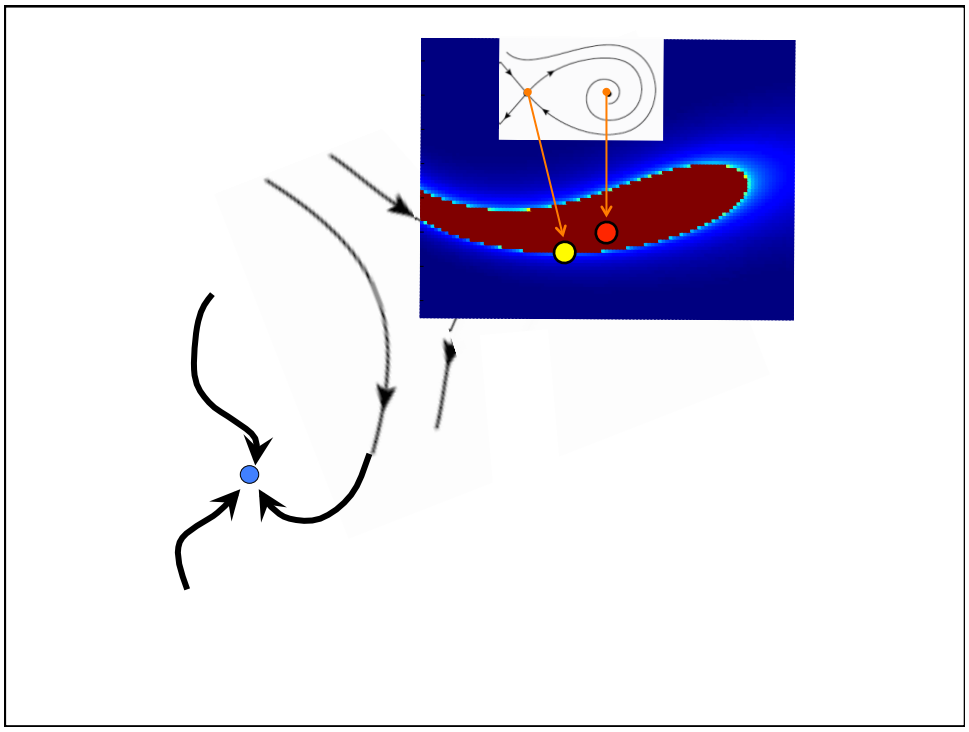
Peculiar Features:

- Turbulence where the laminar profile is stable
- No states of intermediate complexity (like Rayleigh-Benard or Taylor-Couette)
- Turbulent state frequently transient
- Complex spatio-temporal dynamics



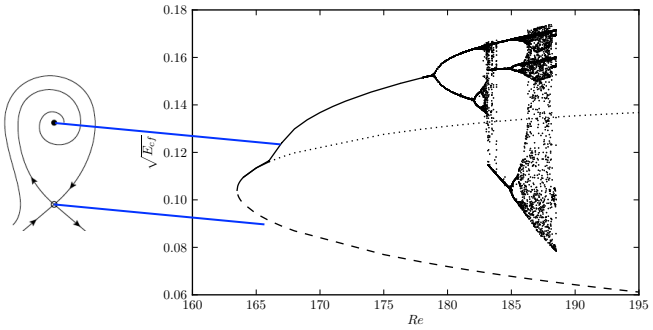




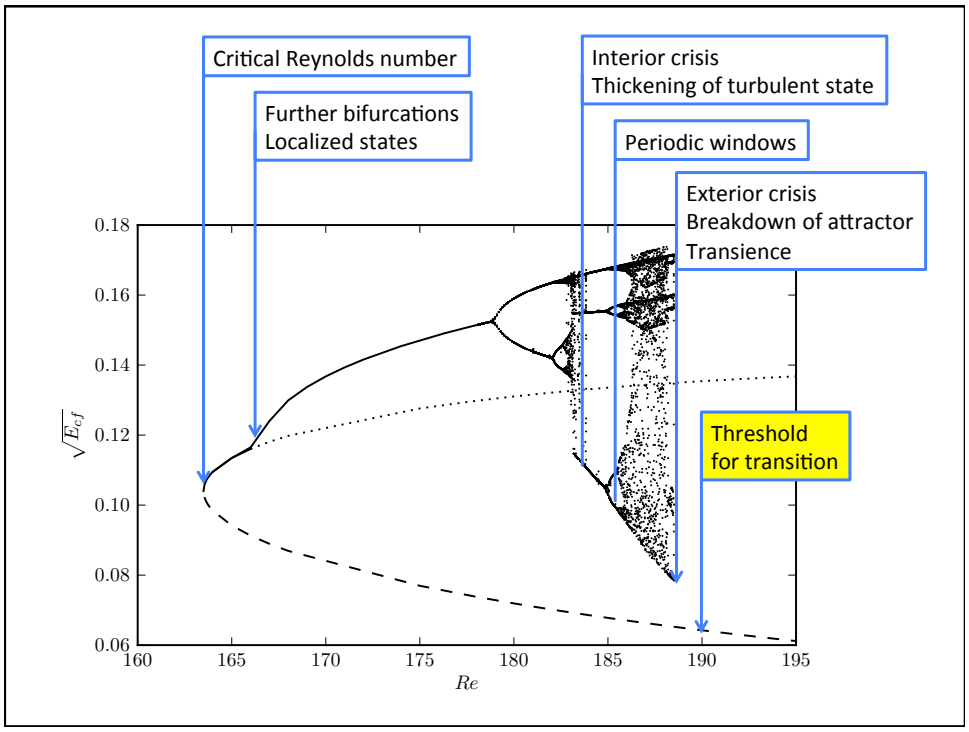


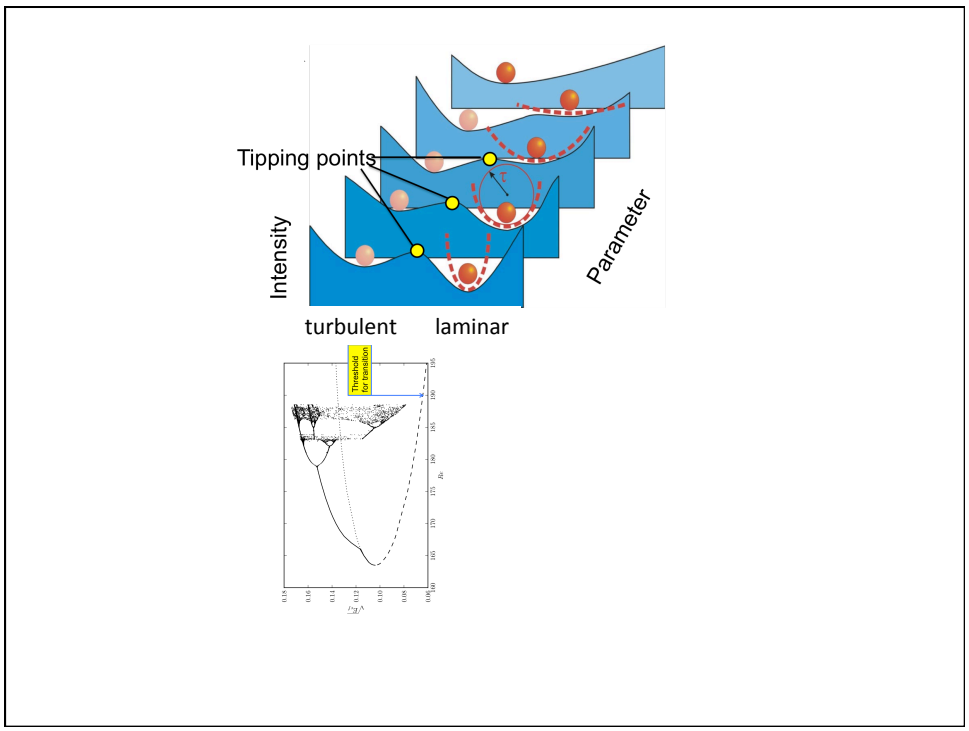
Saddle-node bifurcation and crisis

Plane Couette flow

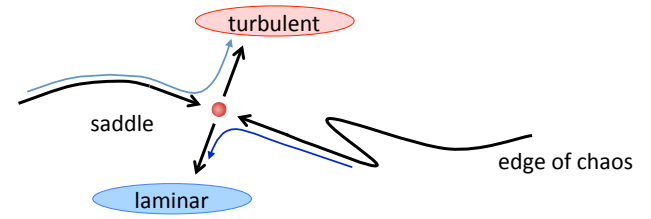


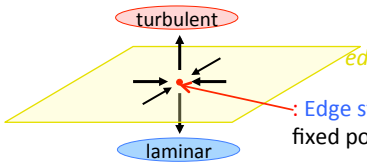
Tobias Kreilos and BE, Chaos **22**, 047505 (2012)


Critical Reynolds number
Further bifurcations
Localized states
Interior crisis
Thickening of turbulent state
Periodic windows
Exterior crisis
Breakdown of attractor
Transience
Threshold
for transition
0.18
0.16
0.14
0.12
0.10
0.08
0.06
160
165
170
175
180
185
190
195
Re


Tipping points
Intensity
Parameter
turbulent
laminar
Threshold
for transition

How to find edge states I:





edge of chaos: stable manifold (co-dimension =1)

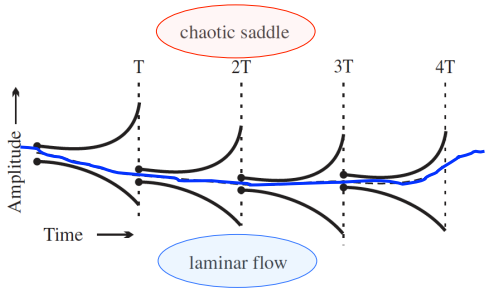
: Edge state:
fixed point (exactly one unstable direction)

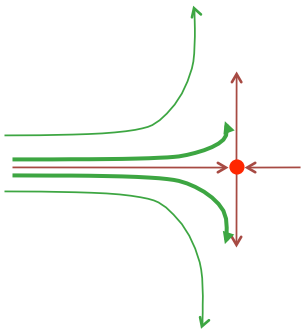
Skufca, Yorke & Eckhardt *PRL* 2006; Schneider, Eckhardt & Yorke *PRL* 2007; Schneider et al. *PRE* 2008

How to find edge states II:

Idea: approximate edge trajectory by pairs of trajectories where one decays and the other becomes turbulent

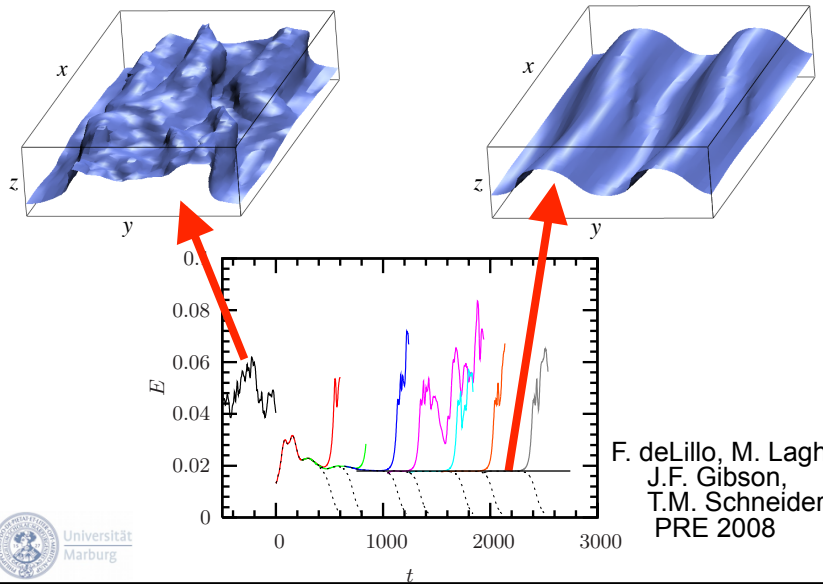
Stay close via successive refinements





Skufca, Yorke & Eckhardt *PRL* 2006

Structures in small pCf domains…



F. deLillo, M. Lagha, J.F. Gibson, T.M. Schneider, BE, PRE 2008

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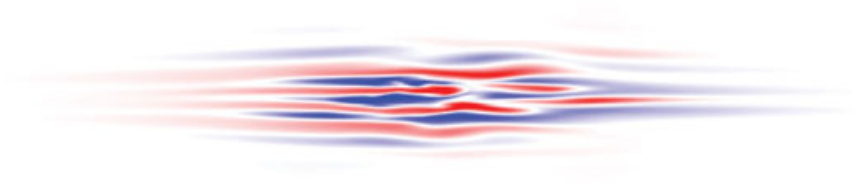
Localized states in wide systems



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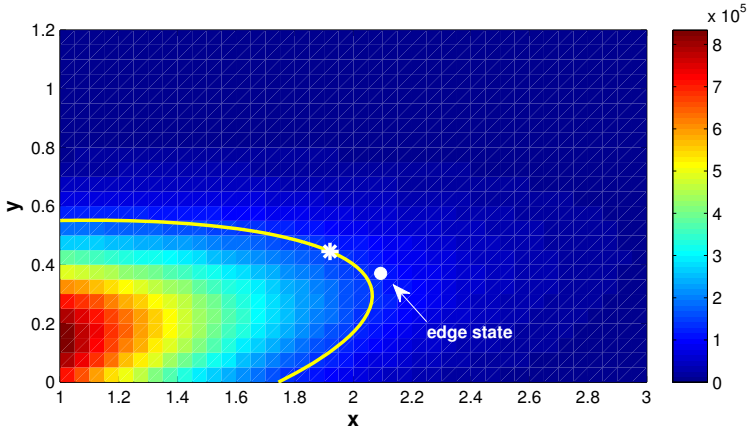
REVIEW ARTICLE

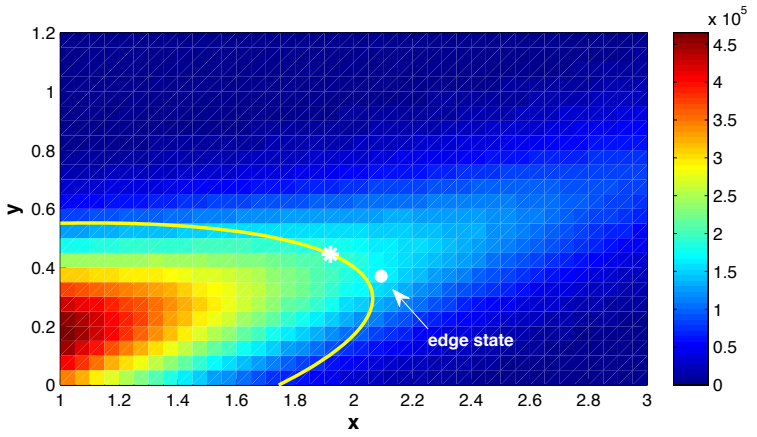
Numerical Bifurcation Methods and their Application to Fluid Dynamics: Analysis beyond Simulation

Henk A. Dijkstra[1,*], Fred W. Wubs[2], Andrew K. Cliffe[3], Eusebius Doedel[4], Ioana F. Dragomirescu[5], Bruno Eckhardt[6], Alexander Yu. Gelfgat[7], Andrew L. Hazel[8], Valerio Lucarini[9,10], Andy G. Salinger[11], Erik T. Phipps[11], Juan Sanchez-Umbria[12], Henk Schuttelaars[13], Laurette S. Tuckerman[14] and Uwe Thiele[15]

Noise, w/o transition



Noise, w/transition



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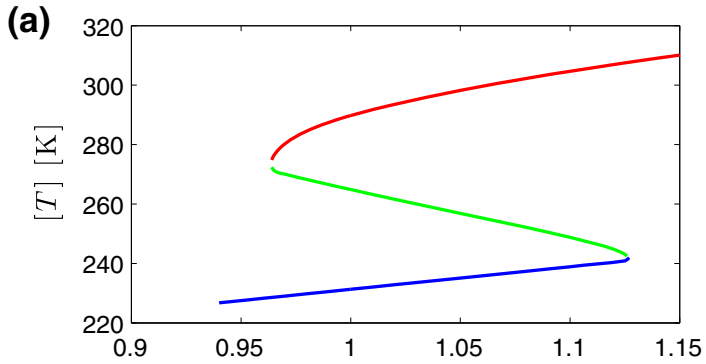
Clim Dyn
DOI 10.1007/s00382-014-2206-5

Global instability in the Ghil–Sellers model

Tamás Bódai · Valerio Lucarini · Frank Lunkeit · Robert Boschi



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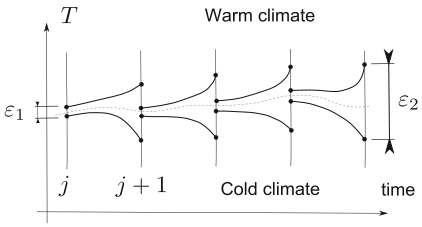

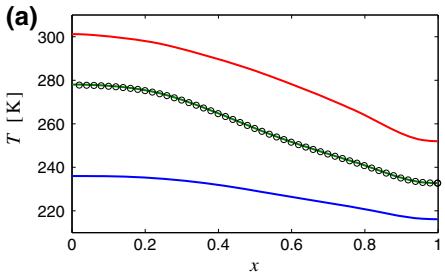

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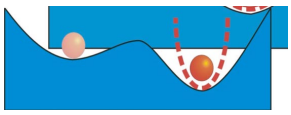
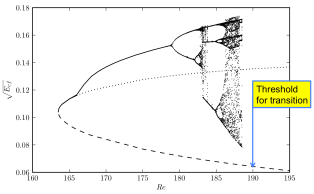
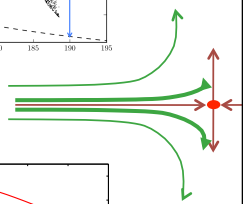
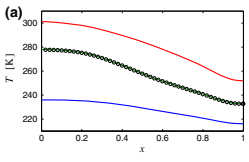
Summary

- Tipping points in climate systems
- Tipping points in transition to turbulence
- Edge tracking for the identification of tipping points
- Tipping points in the Ghil-Sellers climate model









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